Mathematica 11.3 Integration Test Results

Test results for the 286 problems in "1.1.3.3 (a+b x^n)^p (c+d x^n)^q.m"

Problem 30: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(c+d\,x^3\right)^{4/3}}\,dx$$

Optimal (type 2, 16 leaves, 1 step):

$$\frac{x}{c\,\left(c+d\,x^3\right)^{1/3}}$$

Result (type 5, 674 leaves):

$$\left(i\,\sqrt{\frac{\pi}{3}}\,\left(\frac{(-1)^{2/3}\,c^{1/3}}{d^{1/3}}+x\right)\,\left(\frac{c^{1/3}+(-1)^{2/3}\,d^{1/3}\,x}{\left(1+(-1)^{1/3}\right)\,c^{1/3}}\right)^{4/3}\,\left(1+\frac{d^{1/3}\,x}{c^{1/3}}\right)\,\text{Gamma}\left[\frac{1}{3}\right]\right.$$

$$\left(48\,\left(4\,c+2\,\left(2-i\,\sqrt{3}\right)\,c^{2/3}\,d^{1/3}\,x+2\,\left(3+i\,\sqrt{3}\right)\,c^{1/3}\,d^{2/3}\,x^2+3\,\left(1+i\,\sqrt{3}\right)\,d\,x^3\right)\right.$$

$$\text{Hypergeometric2F1}\left[1,\frac{4}{3},\frac{8}{3},\frac{6\,\left(\left(1+i\,\sqrt{3}\right)\,c^{1/3}+\left(1-i\,\sqrt{3}\right)\,d^{1/3}\,x\right)}{\left(3\,i+\sqrt{3}\right)\,\left(\left(3\,i+\sqrt{3}\right)\,c^{1/3}-2\,\sqrt{3}\,d^{1/3}\,x\right)}\right]-$$

$$12\,i\,\left(c^{1/3}+d^{1/3}\,x\right)\,\left(\left(-3\,i+7\,\sqrt{3}\right)\,c^{2/3}+2\,\left(-9\,i+2\,\sqrt{3}\right)\,c^{1/3}\,d^{1/3}\,x-9\,\left(i+\sqrt{3}\right)\,d^{2/3}\,x^2\right)$$

$$\text{Hypergeometric2F1}\left[2,\frac{7}{3},\frac{11}{3},\frac{6\,\left(\left(1+i\,\sqrt{3}\right)\,c^{1/3}+\left(1-i\,\sqrt{3}\right)\,d^{1/3}\,x\right)}{\left(3\,i+\sqrt{3}\right)\,\left(\left(3\,i+\sqrt{3}\right)\,c^{1/3}-2\,\sqrt{3}\,d^{1/3}\,x\right)}\right]-$$

$$36\,i\,\left(c^{1/3}+d^{1/3}\,x\right)^2\,\left(\left(-i+\sqrt{3}\right)\,c^{1/3}-\left(i+\sqrt{3}\right)\,d^{1/3}\,x\right)$$

$$\left.\left.\text{HypergeometricPFQ}\left[\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{11}{3}\right\},\frac{6\,\left(\left(1+i\,\sqrt{3}\right)\,c^{1/3}+\left(1-i\,\sqrt{3}\right)\,d^{1/3}\,x\right)}{\left(3\,i+\sqrt{3}\right)\,\left(\left(3\,i+\sqrt{3}\right)\,c^{1/3}-2\,\sqrt{3}\,d^{1/3}\,x\right)}\right]\right)\right)\Big/$$

$$\left(40\times 2^{1/3}\,\left(3\,i+\sqrt{3}\right)\,c^{2/3}\,\left(\left(3\,i+\sqrt{3}\right)\,c^{1/3}-2\,\sqrt{3}\,d^{1/3}\,x\right)\,\left(c+d\,x^3\right)^{4/3}\right.$$

$$\left.\left(1+\frac{i\,\left((-1)^{2/3}\,c^{1/3}+d^{1/3}\,x\right)}{\sqrt{3}\,c^{1/3}}\right)^{4/3}\,\text{Gamma}\left[\frac{2}{3}\right]\,\text{Gamma}\left[\frac{7}{6}\right]\right)$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \left(a+b x^3\right)^m \left(c+d x^3\right)^p \, dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$x \left(a+b x^3\right)^m \left(1+\frac{b x^3}{a}\right)^{-m} \left(c+d x^3\right)^p \left(1+\frac{d x^3}{c}\right)^{-p} \text{AppellF1}\left[\frac{1}{3},-m,-p,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]$$

Result (type 6, 172 leaves):

$$\left(4 a c x \left(a+b x^3\right)^m \left(c+d x^3\right)^p \text{AppellF1}\left[\frac{1}{3},-m,-p,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right) \Big/ \left(4 a c \, \text{AppellF1}\left[\frac{1}{3},-m,-p,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+3 x^3 \left(b c m \, \text{AppellF1}\left[\frac{4}{3},1-m,-p,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]+a d p \, \text{AppellF1}\left[\frac{4}{3},-m,1-p,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^q}{a+b x^3} \, dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x \left(c+d x^3\right)^q \left(1+\frac{d x^3}{c}\right)^{-q} \text{AppellF1}\left[\frac{1}{3},1,-q,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a}$$

Result (type 6, 162 leaves):

$$\left(4 a c x \left(c+d x^3\right)^q \text{AppellF1}\left[\frac{1}{3},-q,1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right) \Big/ \left(\left(a+b x^3\right) \left(4 a c \, \text{AppellF1}\left[\frac{1}{3},-q,1,\frac{4}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]+3 x^3 \left(a d q \, \text{AppellF1}\left[\frac{4}{3},1-q,1,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]-b c \, \text{AppellF1}\left[\frac{4}{3},-q,2,\frac{7}{3},-\frac{d x^3}{c},-\frac{b x^3}{a}\right]\right)\right)\right)$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^3\right)^q}{\left(a+b x^3\right)^2} \, dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x \left(c+d x^3\right)^q \left(1+\frac{d x^3}{c}\right)^{-q} \text{AppellF1}\left[\frac{1}{3},2,-q,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{a^2}$$

Result (type 6, 162 leaves):

$$\left(4\, a\, c\, x\, \left(c+d\, x^3\right)^q \text{AppellF1}\left[\frac{1}{3},2,-q,\frac{4}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]\right)\Big/\left(\left(a+b\, x^3\right)^2 \left(4\, a\, c\, \text{AppellF1}\left[\frac{1}{3},2,-q,\frac{4}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]+3\, x^3 \left(a\, d\, q\, \text{AppellF1}\left[\frac{4}{3},2,1-q,\frac{7}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]-2\, b\, c\, \text{AppellF1}\left[\frac{4}{3},3,-q,\frac{7}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]\right)\right)\right)$$

Problem 42: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(a+b\, x^3\right)^m dx$$

Optimal (type 5, 44 leaves, 2 steps):

$$x \left(a+b\, x^3\right)^m \left(1+\frac{b\, x^3}{a}\right)^{-m} \text{Hypergeometric2F1}\left[\frac{1}{3},-m,\frac{4}{3},-\frac{b\, x^3}{a}\right]$$

Result (type 6, 196 leaves):

$$\frac{1}{b^{1/3}\,(1+m)} 2^{-m} \left(\left(-1\right)^{2/3} a^{1/3}+b^{1/3}\, x\right) \left(\frac{a^{1/3}+\left(-1\right)^{2/3} b^{1/3}\, x}{\left(1+\left(-1\right)^{1/3}\right) a^{1/3}}\right)^{-m} \left(\frac{i \left(1+\frac{b^{1/3}\, x}{a^{1/3}}\right)}{3\, i+\sqrt{3}}\right)^{-m} \left(a+b\, x^3\right)^m \text{AppellF1}\left[1+m,-m,-m,2+m,-\frac{i \left(\left(-1\right)^{2/3} a^{1/3}+b^{1/3}\, x\right)}{\sqrt{3}\, a^{1/3}},\frac{i+\sqrt{3}-\frac{2\, i\, b^{1/3}\, x}{a^{1/3}}}{3\, i+\sqrt{3}}\right]$$

Problem 43: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b\, x^3\right)^m}{c+d\, x^3} dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x \left(a+b\, x^3\right)^m \left(1+\frac{b\, x^3}{a}\right)^{-m} \text{AppellF1}\left[\frac{1}{3},-m,1,\frac{4}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]}{c}$$

Result (type 6, 162 leaves):

$$-\left(\left(4\, a\, c\, x\, \left(a+b\, x^3\right)^m \text{AppellF1}\left[\frac{1}{3},-m,1,\frac{4}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]\right)\Big/\left(\left(c+d\, x^3\right) \left(-4\, a\, c\, \text{AppellF1}\left[\frac{1}{3},-m,1,\frac{4}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]+3\, x^3 \left(-b\, c\, m\, \text{AppellF1}\left[\frac{4}{3},1-m,1,\frac{7}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]+a\, d\, \text{AppellF1}\left[\frac{4}{3},-m,2,\frac{7}{3},-\frac{b\, x^3}{a},-\frac{d\, x^3}{c}\right]\right)\right)\right)\right)$$

Problem 44: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^m}{\left(c+d x^3\right)^2} \, dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^m \left(1+\frac{b x^3}{a}\right)^{-m} \text{AppellF1}\left[\frac{1}{3},-m,2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^2}$$

Result (type 6, 162 leaves):

$$-\left(\left(4 a c x \left(a+b x^3\right)^m \text{AppellF1}\left[\frac{1}{3},-m,2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/ \left(\left(c+d x^3\right)^2 \left(-4 a c \text{AppellF1}\left[\frac{1}{3},-m,2,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]-3 x^3 \left(b c m \text{AppellF1}\left[\frac{4}{3},1-m,2,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]-2 a d \text{AppellF1}\left[\frac{4}{3},-m,3,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)$$

Problem 45: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(a+b x^3\right)^m}{\left(c+d x^3\right)^3} \, dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x \left(a+b x^3\right)^m \left(1+\frac{b x^3}{a}\right)^{-m} \text{AppellF1}\left[\frac{1}{3},-m,3,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]}{c^3}$$

Result (type 6, 162 leaves):

$$-\left(\left(4 a c x \left(a+b x^3\right)^m \text{AppellF1}\left[\frac{1}{3},-m,3,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\Big/ \left(\left(c+d x^3\right)^3 \left(-4 a c \text{AppellF1}\left[\frac{1}{3},-m,3,\frac{4}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]-3 x^3 \left(b c m \text{AppellF1}\left[\frac{4}{3},1-m,3,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]-3 a d \text{AppellF1}\left[\frac{4}{3},-m,4,\frac{7}{3},-\frac{b x^3}{a},-\frac{d x^3}{c}\right]\right)\right)\right)\right)$$

Problem 46: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left(a+b x^3\right)^{-1-\frac{b c}{3 b c-3 a d}} \left(c+d x^3\right)^{-1+\frac{a d}{3 b c-3 a d}} \, dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\frac{x \left(a+b x^3\right)^{-\frac{b c}{3 b c-3 a d}} \left(c+d x^3\right)^{\frac{a d}{3 b c-3 a d}}}{a c}$$

Result (type 6, 594 leaves) :

$$4\,a\,c\,x\,\left(a+b\,x^3\right)^{\frac{b\,c}{-3\,b\,c+3\,a\,d}}\,\left(c+d\,x^3\right)^{\frac{a\,d}{3\,b\,c-3\,a\,d}}$$
$$\left(\left(d\,\text{AppellF1}\left[\frac{1}{3},\frac{b\,c}{3\,b\,c-3\,a\,d},1+\frac{a\,d}{-3\,b\,c+3\,a\,d},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right/$$
$$\left(\left(c+d\,x^3\right)\,\left(4\,a\,c\,\left(-b\,c+a\,d\right)\,\text{AppellF1}\left[\frac{1}{3},\frac{b\,c}{3\,b\,c-3\,a\,d},1+\frac{a\,d}{-3\,b\,c+3\,a\,d},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.\right.$$
$$x^3\,\left(a\,d\,\left(3\,b\,c-4\,a\,d\right)\,\text{AppellF1}\left[\frac{4}{3},\frac{b\,c}{3\,b\,c-3\,a\,d},2+\frac{a\,d}{-3\,b\,c+3\,a\,d},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.$$
$$\left.\left.\left.b^2\,c^2\,\text{AppellF1}\left[\frac{4}{3},1+\frac{b\,c}{3\,b\,c-3\,a\,d},1+\frac{a\,d}{-3\,b\,c+3\,a\,d},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)+$$
$$\left(b\,\text{AppellF1}\left[\frac{1}{3},1+\frac{b\,c}{3\,b\,c-3\,a\,d},\frac{a\,d}{-3\,b\,c+3\,a\,d},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\Big/$$
$$\left(\left(a+b\,x^3\right)\,\left(4\,a\,c\,\left(b\,c-a\,d\right)\,\text{AppellF1}\left[\frac{1}{3},1+\frac{b\,c}{3\,b\,c-3\,a\,d},\frac{a\,d}{-3\,b\,c+3\,a\,d},\frac{4}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.\right.$$
$$x^3\,\left(a^2\,d^2\,\text{AppellF1}\left[\frac{4}{3},1+\frac{b\,c}{3\,b\,c-3\,a\,d},1+\frac{a\,d}{-3\,b\,c+3\,a\,d},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]+\right.$$
$$\left.\left.\left.\left.b\,c\,\left(-4\,b\,c+3\,a\,d\right)\,\text{AppellF1}\left[\frac{4}{3},2+\frac{b\,c}{3\,b\,c-3\,a\,d},\frac{a\,d}{-3\,b\,c+3\,a\,d},\frac{7}{3},-\frac{b\,x^3}{a},-\frac{d\,x^3}{c}\right]\right)\right)\right)\right)$$

Problem 74: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b\,x^4\right)^{5/2}}{c-d\,x^4}\,dx$$

Optimal (type 4, 321 leaves, 10 steps) :

$$-\frac{b\,\left(7\,b\,c-13\,a\,d\right)\,x\,\sqrt{a-b\,x^4}}{21\,d^2}+\frac{b\,x\,\left(a-b\,x^4\right)^{3/2}}{7\,d}+\frac{1}{21\,d^3\,\sqrt{a-b\,x^4}}$$
$$a^{1/4}\,b^{3/4}\,\left(21\,b^2\,c^2-56\,a\,b\,c\,d+47\,a^2\,d^2\right)\,\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]-$$
$$\frac{a^{1/4}\,\left(b\,c-a\,d\right)^3\,\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,d^3\,\sqrt{a-b\,x^4}}-$$
$$\frac{a^{1/4}\,\left(b\,c-a\,d\right)^3\,\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,d^3\,\sqrt{a-b\,x^4}}$$

Result (type 6, 385 leaves) :

$$\frac{1}{105\, d^2 \sqrt{a - b x^4}} x \left(5 b \left(-a + b x^4\right) \left(7 b c - 16 a d + 3 b d x^4\right) + \left(25 a^2 c \left(7 b^2 c^2 - 16 a b c d + 21 a^2 d^2\right) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right) \Big/ \left(\left(c - d x^4\right) \left(5 a c\, \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right] + 2 x^4 \left(2 a d\, \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right] + b c\, \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right)\right)\right) - \left(9 a b c \left(21 b^2 c^2 - 56 a b c d + 47 a^2 d^2\right) x^4 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right) \Big/ \left(\left(c - d x^4\right) \left(9 a c\, \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right] + 2 x^4 \left(2 a d\, \text{AppellF1}\left[\frac{9}{4}, \frac{1}{2}, 2, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right] + b c\, \text{AppellF1}\left[\frac{9}{4}, \frac{3}{2}, 1, \frac{13}{4}, \frac{b x^4}{a}, \frac{d x^4}{c}\right]\right)\right)\right)\right)$$

Problem 75: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a - b x^4\right)^{3/2}}{c - d x^4} \, dx$$

Optimal (type 4, 277 leaves, 9 steps):

$$\frac{b x \sqrt{a - b x^4}}{3 d} - \frac{a^{1/4} b^{3/4} (3 b c - 5 a d) \sqrt{1 - \frac{b x^4}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{3 d^2 \sqrt{a - b x^4}} + \frac{a^{1/4} (b c - a d)^2 \sqrt{1 - \frac{b x^4}{a}}\, \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{2 b^{1/4} c d^2 \sqrt{a - b x^4}} + \frac{a^{1/4} (b c - a d)^2 \sqrt{1 - \frac{b x^4}{a}}\, \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right], -1\right]}{2 b^{1/4} c d^2 \sqrt{a - b x^4}}$$

Result (type 6, 419 leaves):

$$\left(x\left(-\left(\left(25 a^2 c\,(-b c+3 a d)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(5 a c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)+\left(b\left(-9 a c\left(-2 b c x^4+5 b d x^8+5 a\left(c-2 d x^4\right)\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]-10 x^4\left(a-b x^4\right)\left(c-d x^4\right)\left(2 a d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\Big/\left(9 a c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\right)\Big/\left(15 d\sqrt{a-b x^4}\left(-c+d x^4\right)\right)$$

Problem 76: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a-b x^4}}{c-d x^4}\,dx$$

Optimal (type 4, 240 leaves, 8 steps):

$$\frac{a^{1/4} b^{3/4}\sqrt{1-\frac{b x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{d\sqrt{a-b x^4}}-\frac{a^{1/4}(b c-a d)\sqrt{1-\frac{b x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{2 b^{1/4} c d\sqrt{a-b x^4}}-\frac{a^{1/4}(b c-a d)\sqrt{1-\frac{b x^4}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{2 b^{1/4} c d\sqrt{a-b x^4}}$$

Result (type 6, 155 leaves):

$$-\left(\left(5 a c x\sqrt{a-b x^4}\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(\left(c-d x^4\right)\left(-5 a c\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(-2 a d\,\text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\right)$$

Problem 77: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a-b x^4}\,\left(c-d x^4\right)}\,dx$$

Optimal (type 4, 162 leaves, 5 steps):

$$\frac{a^{1/4}\sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\ -1\right]}{2 b^{1/4} c \sqrt{a-b x^4}}+\frac{a^{1/4}\sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\ -1\right]}{2 b^{1/4} c \sqrt{a-b x^4}}$$

Result (type 6, 156 leaves):

$$-\left(\left(5\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(\sqrt{a-b x^4}\,\left(-c+d x^4\right)\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\right)$$

Problem 78: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^4\right)^{3/2}\left(c-d x^4\right)}\,dx$$

Optimal (type 4, 281 leaves, 9 steps):

$$\frac{b x}{2 a (b c-a d)\sqrt{a-b x^4}}+\frac{b^{3/4}\sqrt{1-\frac{b x^4}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{2 a^{3/4}(b c-a d)\sqrt{a-b x^4}}-\frac{a^{1/4} d\sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{2 b^{1/4} c (b c-a d)\sqrt{a-b x^4}}-\frac{a^{1/4} d\sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\ \text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{2 b^{1/4} c (b c-a d)\sqrt{a-b x^4}}$$

Result (type 6, 329 leaves):

$$\left(x\left(-\frac{5 b}{a}-\left(25 c (b c-2 a d) \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(\left(c-d x^4\right)\left(5 a c\, \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d\, \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\, \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)+\left(9 b c d x^4 \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(\left(c-d x^4\right)\left(9 a c\, \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d\, \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\, \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(10 (-b c+a d)\sqrt{a-b x^4}\right)$$

Problem 79: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a-b x^4\right)^{5/2}\left(c-d x^4\right)} \, dx$$

Optimal (type 4, 334 leaves, 10 steps):

$$\frac{b x}{6 a (b c-a d)\left(a-b x^4\right)^{3/2}}+\frac{b (5 b c-11 a d) x}{12 a^2 (b c-a d)^2 \sqrt{a-b x^4}}+\frac{b^{3/4}(5 b c-11 a d)\sqrt{1-\frac{b x^4}{a}}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{12 a^{7/4}(b c-a d)^2\sqrt{a-b x^4}}+\frac{a^{1/4} d^2\sqrt{1-\frac{b x^4}{a}}\, \text{EllipticPi}\left[-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{2 b^{1/4} c (b c-a d)^2\sqrt{a-b x^4}}+\frac{a^{1/4} d^2\sqrt{1-\frac{b x^4}{a}}\, \text{EllipticPi}\left[\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{2 b^{1/4} c (b c-a d)^2\sqrt{a-b x^4}}$$

Result (type 6, 396 leaves):

$$\left(x\left(\frac{5\,b\left(13\,a^2\,d+5\,b^2\,c\,x^4-a\,b\left(7\,c+11\,d\,x^4\right)\right)}{-a+b\,x^4}+\right.\right.$$
$$\left(25\,a\,c\left(5\,b^2\,c^2-11\,a\,b\,c\,d+12\,a^2\,d^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/$$
$$\left(\left(c-d\,x^4\right)\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\right.\right.\right.$$
$$\left.\left.\left.\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)+$$
$$\left(9\,a\,b\,c\,d\left(-5\,b\,c+11\,a\,d\right)x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/$$
$$\left(\left(c-d\,x^4\right)\left(9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+\right.\right.$$
$$2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+\right.$$
$$\left.\left.\left.\left.\left.b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(60\,a^2\left(b\,c-a\,d\right)^2\sqrt{a-b\,x^4}\right)$$

Problem 80: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^4\right)^{3/2}}{c+d\,x^4}\,dx$$

Optimal (type 4, 926 leaves, 10 steps):

$$\frac{b\,x\,\sqrt{a+b\,x^4}}{3\,d}-\frac{\left(b\,c-a\,d\right)^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{b\,c-a\,d}\;x}{(-c)^{1/4}\,d^{1/4}\,\sqrt{a+b\,x^4}}\right]}{4\,(-c)^{3/4}\,d^{7/4}}-\frac{\left(-b\,c+a\,d\right)^{3/2}\,\text{ArcTan}\left[\frac{\sqrt{-b\,c+a\,d}\;x}{(-c)^{1/4}\,d^{1/4}\,\sqrt{a+b\,x^4}}\right]}{4\,(-c)^{3/4}\,d^{7/4}}-$$

$$\left(b^{3/4}\,\left(3\,b\,c-5\,a\,d\right)\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$

$$\left(6\,a^{1/4}\,d^2\,\sqrt{a+b\,x^4}\right)+\left(b^{1/4}\,\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)\,\left(b\,c-a\,d\right)^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4\,a^{1/4}\,\sqrt{-c}\,d^2\,\left(b\,c+a\,d\right)\,\sqrt{a+b\,x^4}\right)+$$

$$\left(b^{1/4}\,\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)\,\left(b\,c-a\,d\right)^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4\,a^{1/4}\,\sqrt{-c}\,d^2\,\left(b\,c+a\,d\right)\,\sqrt{a+b\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2\,\left(b\,c-a\,d\right)^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.-\frac{\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2}{4\,\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8\,a^{1/4}\,b^{1/4}\,c\,d^2\,\left(b\,c+a\,d\right)\,\sqrt{a+b\,x^4}\right)+$$

$$\left(\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2\,\left(b\,c-a\,d\right)^2\,\left(\sqrt{a}+\sqrt{b}\,x^2\right)\,\sqrt{\frac{a+b\,x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\;\text{EllipticPi}\left[\right.\right.$$

$$\left.\left.\frac{\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2}{4\,\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}},2\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8\,a^{1/4}\,b^{1/4}\,c\,d^2\,\left(b\,c+a\,d\right)\,\sqrt{a+b\,x^4}\right)$$

Result (type 6, 435 leaves) :

$$\left(x\left(\left(25\,a^2\,c\,\left(-b\,c+3\,a\,d\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]-2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)+\left(b\left(-9\,a\,c\,\left(5\,a\,\left(c+2\,d\,x^4\right)+b\,x^4\,\left(2\,c+5\,d\,x^4\right)\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+10\,x^4\,\left(a+b\,x^4\right)\,\left(c+d\,x^4\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\Big/\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)\Big/\left(15\,d\,\sqrt{a+b\,x^4}\,\left(c+d\,x^4\right)\right)$$

Problem 81: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+b\,x^4}}{c+d\,x^4}\,dx$$

Optimal (type 4, 881 leaves, 9 steps):

$$\frac{\sqrt{b c-a d}\ \text{ArcTan}\left[\frac{\sqrt{b c-a d}\ x}{(-c)^{1/4} d^{1/4} \sqrt{a+b x^4}}\right]}{4 (-c)^{3/4} d^{3/4}}-\frac{\sqrt{-b c+a d}\ \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\ x}{(-c)^{1/4} d^{1/4} \sqrt{a+b x^4}}\right]}{4 (-c)^{3/4} d^{3/4}}+$$

$$\frac{b^{3/4} \left(\sqrt{a}+\sqrt{b}\ x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]}{2 a^{1/4} d \sqrt{a+b x^4}}-$$

$$\left(b^{1/4} \left(\sqrt{b} \sqrt{-c}-\sqrt{a} \sqrt{d}\right) (b c-a d) \left(\sqrt{a}+\sqrt{b}\ x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(4 a^{1/4} \sqrt{-c}\ d (b c+a d) \sqrt{a+b x^4}\right)-$$

$$\left(b^{1/4} \left(\sqrt{b}+\frac{\sqrt{a} \sqrt{d}}{\sqrt{-c}}\right) (b c-a d) \left(\sqrt{a}+\sqrt{b}\ x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(4 a^{1/4} d (b c+a d) \sqrt{a+b x^4}\right)-$$

$$\left(\left(\sqrt{b} \sqrt{-c}+\sqrt{a} \sqrt{d}\right)^2 (b c-a d) \left(\sqrt{a}+\sqrt{b}\ x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{-c}-\sqrt{a} \sqrt{d}\right)^2}{4 \sqrt{a} \sqrt{b} \sqrt{-c} \sqrt{d}},2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(8 a^{1/4} b^{1/4} c d (b c+a d) \sqrt{a+b x^4}\right)-$$

$$\left(\left(\sqrt{b} \sqrt{-c}-\sqrt{a} \sqrt{d}\right)^2 (b c-a d) \left(\sqrt{a}+\sqrt{b}\ x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\ x^2\right)^2}}\ \text{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{-c}+\sqrt{a} \sqrt{d}\right)^2}{4 \sqrt{a} \sqrt{b} \sqrt{-c} \sqrt{d}},2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(8 a^{1/4} b^{1/4} c d (b c+a d) \sqrt{a+b x^4}\right)$$

Result (type 6, 161 leaves):

$$\left(5 a c x \sqrt{a+b x^4}\ \text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right) \Big/ \left(\left(c+d x^4\right) \left(5 a c\ \text{AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+2 x^4 \left(-2 a d\ \text{AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+b c\ \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)$$

Problem 82: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a+b x^4}\,\left(c+d x^4\right)}\,dx$$

Optimal (type 4, 742 leaves, 7 steps):

$$-\frac{d^{1/4}\,\text{ArcTan}\left[\frac{\sqrt{b c-a d}\;x}{(-c)^{1/4} d^{1/4} \sqrt{a+b x^4}}\right]}{4\,(-c)^{3/4}\sqrt{b c-a d}}-\frac{d^{1/4}\,\text{ArcTan}\left[\frac{\sqrt{-b c+a d}\;x}{(-c)^{1/4} d^{1/4} \sqrt{a+b x^4}}\right]}{4\,(-c)^{3/4}\sqrt{-b c+a d}}+$$
$$\left(b^{1/4}\left(\sqrt{b}+\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/$$
$$\left(4\,a^{1/4}\,(b c+a d)\sqrt{a+b x^4}\right)+\left(b^{1/4}\left(\sqrt{b}\,c+\sqrt{a}\,\sqrt{-c}\,\sqrt{d}\right)\left(\sqrt{a}+\sqrt{b}\,x^2\right)\right.$$
$$\left.\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(4\,a^{1/4}\,c\,(b c+a d)\sqrt{a+b x^4}\right)+$$
$$\left(\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticPi}\left[\right.\right.$$
$$\left.\left.-\frac{\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2}{4\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8\,a^{1/4}\,b^{1/4}\,c\,(b c+a d)\sqrt{a+b x^4}\right)+$$
$$\left(\left(\sqrt{b}\,\sqrt{-c}-\sqrt{a}\,\sqrt{d}\right)^2\left(\sqrt{a}+\sqrt{b}\,x^2\right)\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b}\,x^2\right)^2}}\,\text{EllipticPi}\left[\right.\right.$$
$$\left.\left.\frac{\left(\sqrt{b}\,\sqrt{-c}+\sqrt{a}\,\sqrt{d}\right)^2}{4\sqrt{a}\,\sqrt{b}\,\sqrt{-c}\,\sqrt{d}},\,2\,\text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right)\Big/\left(8\,a^{1/4}\,b^{1/4}\,c\,(b c+a d)\sqrt{a+b x^4}\right)$$

Result (type 6, 161 leaves):

$$-\left(\left(5\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\right.$$
$$\left(\sqrt{a+b x^4}\,\left(c+d x^4\right)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+2\,x^4\left(2\,a\,d\right.\right.\right.$$
$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)\right)$$

Problem 83: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^4\right)^{3/2} \left(c+d x^4\right)} \, dx$$

Optimal (type 4, 913 leaves, 10 steps):

$$\frac{b x}{2 a (b c-a d) \sqrt{a+b x^4}}+\frac{d^{5/4} \text{ArcTan}\left[\frac{\sqrt{b c-a d}\, x}{(-c)^{1/4} d^{1/4} \sqrt{a+b x^4}}\right]}{4 (-c)^{3/4} (b c-a d)^{3/2}}-\frac{d^{5/4} \text{ArcTan}\left[\frac{\sqrt{-b c+a d}\, x}{(-c)^{1/4} d^{1/4} \sqrt{a+b x^4}}\right]}{4 (-c)^{3/4} (-b c+a d)^{3/2}}+$$

$$\frac{b^{3/4} \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]}{4 a^{5/4} (b c-a d) \sqrt{a+b x^4}}-$$

$$\left(b^{1/4} \left(\sqrt{b}+\frac{\sqrt{a} \sqrt{d}}{\sqrt{-c}}\right) d \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(4 a^{1/4} (b c-a d) (b c+a d) \sqrt{a+b x^4}\right)-$$

$$\left(b^{1/4} \left(\sqrt{b} c+\sqrt{a} \sqrt{-c} \sqrt{d}\right) d \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/ \left(4 a^{1/4} c \left(b^2 c^2-a^2 d^2\right) \sqrt{a+b x^4}\right)-$$

$$\left(\left(\sqrt{b} \sqrt{-c}+\sqrt{a} \sqrt{d}\right)^2 d \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}}\right.$$

$$\left.\text{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{-c}-\sqrt{a} \sqrt{d}\right)^2}{4 \sqrt{a} \sqrt{b} \sqrt{-c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 a^{1/4} b^{1/4} c (b c-a d) (b c+a d) \sqrt{a+b x^4}\right)-\left(\left(\sqrt{b} \sqrt{-c}-\sqrt{a} \sqrt{d}\right)^2 d \left(\sqrt{a}+\sqrt{b} x^2\right)\right.$$

$$\left.\sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{-c}+\sqrt{a} \sqrt{d}\right)^2}{4 \sqrt{a} \sqrt{b} \sqrt{-c} \sqrt{d}}, 2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right], \frac{1}{2}\right]\right) \Big/$$

$$\left(8 a^{1/4} b^{1/4} c (b c-a d) (b c+a d) \sqrt{a+b x^4}\right)$$

Result (type 6, 342 leaves):

$$\left(x\left(-\frac{5b}{a}+\left(25c\left(bc-2ad\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\right)\Big/\left(\left(c+dx^4\right)\left(-5ac\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+2x^4\left(2ad\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+bc\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\right)\right)\right)+\left(9bcdx^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\right)\Big/\left(\left(c+dx^4\right)\left(-9ac\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+2x^4\left(2ad\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+bc\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(10\left(-bc+ad\right)\sqrt{a+bx^4}\right)$$

Problem 84: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+bx^4\right)^{5/2}\left(c+dx^4\right)}\,dx$$

Optimal (type 4, 976 leaves, 11 steps) :

$$\frac{b x}{6 a (b c-a d) \left(a+b x^4\right)^{3/2}}+\frac{b (5 b c-11 a d) x}{12 a^2 (b c-a d)^2 \sqrt{a+b x^4}}-\frac{d^{9/4} \text{ArcTan}\left[\frac{\sqrt{b c-a d} x}{(-c)^{1/4} d^{1/4} \sqrt{a+b x^4}}\right]}{4 (-c)^{3/4} (b c-a d)^{5/2}}-\frac{d^{9/4} \text{ArcTan}\left[\frac{\sqrt{-b c+a d} x}{(-c)^{1/4} d^{1/4} \sqrt{a+b x^4}}\right]}{4 (-c)^{3/4} (-b c+a d)^{5/2}}+$$

$$\left(b^{3/4} (5 b c-11 a d) \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(24 a^{9/4} (b c-a d)^2 \sqrt{a+b x^4}\right)+$$

$$\left(b^{1/4} \left(\sqrt{b} c-\sqrt{a} \sqrt{-c} \sqrt{d}\right) d^2 \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(4 a^{1/4} c (b c-a d)^2 (b c+a d) \sqrt{a+b x^4}\right)+$$

$$\left(b^{1/4} \left(\sqrt{b} c+\sqrt{a} \sqrt{-c} \sqrt{d}\right) d^2 \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticF}\left[2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(4 a^{1/4} c (b c-a d)^2 (b c+a d) \sqrt{a+b x^4}\right)+$$

$$\left(\left(\sqrt{b} \sqrt{-c}+\sqrt{a} \sqrt{d}\right)^2 d^2 \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticPi}\left[-\frac{\left(\sqrt{b} \sqrt{-c}-\sqrt{a} \sqrt{d}\right)^2}{4 \sqrt{a} \sqrt{b} \sqrt{-c} \sqrt{d}},2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(8 a^{1/4} b^{1/4} c (b c-a d)^2 (b c+a d) \sqrt{a+b x^4}\right)+$$

$$\left(\left(\sqrt{b} \sqrt{-c}-\sqrt{a} \sqrt{d}\right)^2 d^2 \left(\sqrt{a}+\sqrt{b} x^2\right) \sqrt{\frac{a+b x^4}{\left(\sqrt{a}+\sqrt{b} x^2\right)^2}} \text{EllipticPi}\left[\frac{\left(\sqrt{b} \sqrt{-c}+\sqrt{a} \sqrt{d}\right)^2}{4 \sqrt{a} \sqrt{b} \sqrt{-c} \sqrt{d}},2 \text{ArcTan}\left[\frac{b^{1/4} x}{a^{1/4}}\right],\frac{1}{2}\right]\right) \Big/ \left(8 a^{1/4} b^{1/4} c (b c-a d)^2 (b c+a d) \sqrt{a+b x^4}\right)$$

Result (type 6, 406 leaves) :

$$\left(x\left(\frac{5\,b\,\left(-13\,a^2\,d+5\,b^2\,c\,x^4+a\,b\,\left(7\,c-11\,d\,x^4\right)\right)}{a+b\,x^4}+\left(25\,a\,c\,\left(5\,b^2\,c^2-11\,a\,b\,c\,d+12\,a^2\,d^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]-2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)+\left(9\,a\,b\,c\,d\,\left(-5\,b\,c+11\,a\,d\right)\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(60\,a^2\,\left(b\,c-a\,d\right)^2\,\sqrt{a+b\,x^4}\right)$$

Problem 85: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a-b\,x^4\right)^{7/2}}{\left(c-d\,x^4\right)^2}\,dx$$

Optimal (type 4, 426 leaves, 11 steps):

$$-\frac{b\,\left(77\,b^2\,c^2-122\,a\,b\,c\,d+21\,a^2\,d^2\right)\,x\,\sqrt{a-b\,x^4}}{84\,c\,d^3}+\frac{b\,\left(11\,b\,c-7\,a\,d\right)\,x\,\left(a-b\,x^4\right)^{3/2}}{28\,c\,d^2}-\frac{\left(b\,c-a\,d\right)\,x\,\left(a-b\,x^4\right)^{5/2}}{4\,c\,d\,\left(c-d\,x^4\right)}+\frac{1}{84\,c\,d^4\,\sqrt{a-b\,x^4}}a^{1/4}\,b^{3/4}\left(231\,b^3\,c^3-553\,a\,b^2\,c^2\,d+349\,a^2\,b\,c\,d^2+21\,a^3\,d^3\right)\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]-\left(a^{1/4}\,\left(b\,c-a\,d\right)^3\,\left(11\,b\,c+3\,a\,d\right)\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]\right)\Big/\left(8\,b^{1/4}\,c^2\,d^4\,\sqrt{a-b\,x^4}\right)-\left(a^{1/4}\,\left(b\,c-a\,d\right)^3\,\left(11\,b\,c+3\,a\,d\right)\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]\right)\Big/\left(8\,b^{1/4}\,c^2\,d^4\,\sqrt{a-b\,x^4}\right)$$

Result (type 6, 580 leaves):

$$\frac{1}{420\,d^3\sqrt{a-b\,x^4}\,\left(c-d\,x^4\right)} x\left(\left(25\,a^2\left(77\,b^3\,c^3-155\,a\,b^2\,c^2\,d+63\,a^2\,b\,c\,d^2+63\,a^3\,d^3\right)\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)+\left(9\,a\,c\left(105\,a^4\,d^3+a^2\,b^2\,c\,d\left(775\,c-494\,d\,x^4\right)-63\,a^3\,b\,d^2\left(5\,c+2\,d\,x^4\right)+2\,b^4\,c\,x^4\left(77\,c^2-110\,c\,d\,x^4-30\,d^2\,x^8\right)+a\,b^3\,c\left(-385\,c^2-2\,c\,d\,x^4+520\,d^2\,x^8\right)\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]-10\,x^4\left(-a+b\,x^4\right)\left(-63\,a^2\,b\,c\,d^2+21\,a^3\,d^3+a\,b^2\,c\,d\left(155\,c-92\,d\,x^4\right)+b^3\,c\left(-77\,c^2+44\,c\,d\,x^4+12\,d^2\,x^8\right)\right)\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\Big/\left(c\left(9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)\right)$$

Problem 86: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a-b\,x^4\right)^{5/2}}{\left(c-d\,x^4\right)^2}\,dx$$

Optimal (type 4, 365 leaves, 10 steps):

$$\frac{b\left(7\,b\,c-3\,a\,d\right)x\sqrt{a-b\,x^4}}{12\,c\,d^2}-\frac{\left(b\,c-a\,d\right)x\left(a-b\,x^4\right)^{3/2}}{4\,c\,d\left(c-d\,x^4\right)}-\frac{1}{12\,c\,d^3\sqrt{a-b\,x^4}}a^{1/4}\,b^{3/4}\left(21\,b^2\,c^2-26\,a\,b\,c\,d-3\,a^2\,d^2\right)\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]+\left(a^{1/4}\left(b\,c-a\,d\right)^2\left(7\,b\,c+3\,a\,d\right)\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]\right)\Big/\left(8\,b^{1/4}\,c^2\,d^3\sqrt{a-b\,x^4}\right)+\left(a^{1/4}\left(b\,c-a\,d\right)^2\left(7\,b\,c+3\,a\,d\right)\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]\right)\Big/\left(8\,b^{1/4}\,c^2\,d^3\sqrt{a-b\,x^4}\right)$$

Result (type 6, 491 leaves):

$$\left(x\left(-\left(\left(25 a^2\left(-7 b^2 c^2+6 a b c d+9 a^2 d^2\right) \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(5 a c \,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)+\left(-9 a c\left(15 a^3 d^2+a b^2 c\left(35 c-16 d x^4\right)-6 a^2 b d\left(5 c+3 d x^4\right)+2 b^3 c x^4\left(-7 c+10 d x^4\right)\right)\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]-10 x^4\left(a-b x^4\right)\left(-6 a b c d+3 a^2 d^2+b^2 c\left(7 c-4 d x^4\right)\right)\left(2 a d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\Big/\left(c\left(9 a c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(60 d^2\sqrt{a-b x^4}\left(-c+d x^4\right)\right)$$

Problem 87: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a-b x^4\right)^{3/2}}{\left(c-d x^4\right)^2}\,dx$$

Optimal (type 4, 309 leaves, 9 steps):

$$-\frac{(b c-a d)\, x\sqrt{a-b x^4}}{4 c d\left(c-d x^4\right)}+\frac{a^{1/4} b^{3/4}(3 b c+a d)\sqrt{1-\frac{b x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{4 c d^2\sqrt{a-b x^4}}-\left(3 a^{1/4}(b c-a d)(b c+a d)\sqrt{1-\frac{b x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]\right)\Big/\left(8 b^{1/4} c^2 d^2\sqrt{a-b x^4}\right)-\left(3 a^{1/4}(b c-a d)(b c+a d)\sqrt{1-\frac{b x^4}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]\right)\Big/\left(8 b^{1/4} c^2 d^2\sqrt{a-b x^4}\right)$$

Result (type 6, 423 leaves):

$$\left(x\left(-\left(\left(25 a^2 (b c+3 a d) \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(5 a c \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)+\left(-9 a c \left(5 a^2 d+2 b^2 c x^4-a b\left(5 c+6 d x^4\right)\right) \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]-10(-b c+a d) x^4\left(a-b x^4\right)\left(2 a d \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\Big/\left(c\left(9 a c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(20 d \sqrt{a-b x^4}\left(-c+d x^4\right)\right)$$

Problem 88: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a-b x^4}}{\left(c-d x^4\right)^2}\, dx$$

Optimal (type 4, 276 leaves, 9 steps):

$$\frac{x \sqrt{a-b x^4}}{4 c \left(c-d x^4\right)}+\frac{a^{1/4} b^{3/4} \sqrt{1-\frac{b x^4}{a}}\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{4 c d \sqrt{a-b x^4}}-\frac{a^{1/4} (b c-3 a d) \sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 d \sqrt{a-b x^4}}-\frac{a^{1/4} (b c-3 a d) \sqrt{1-\frac{b x^4}{a}}\ \text{EllipticPi}\left[\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 d \sqrt{a-b x^4}}$$

Result (type 6, 310 leaves):

$$\left(x\left(-\frac{5\left(a-b x^4\right)}{c}-\left(75 a^2 \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(5 a c \text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)+\left(9 a b x^4 \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\Big/\left(9 a c \text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+2 x^4\left(2 a d \text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]+b c \text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b x^4}{a},\frac{d x^4}{c}\right]\right)\right)\right)\right)\Big/\left(20\sqrt{a-b x^4}\left(-c+d x^4\right)\right)$$

Problem 89: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a-b x^4}\left(c-d x^4\right)^2}\,dx$$

Optimal (type 4, 310 leaves, 9 steps):

$$-\frac{d x \sqrt{a-b x^4}}{4 c (b c-a d)\left(c-d x^4\right)}-\frac{a^{1/4} b^{3/4}\sqrt{1-\frac{b x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{4 c (b c-a d)\sqrt{a-b x^4}}+\left(a^{1/4}(5 b c-3 a d)\sqrt{1-\frac{b x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]\right)\Big/\left(8 b^{1/4} c^2 (b c-a d)\sqrt{a-b x^4}\right)+\frac{a^{1/4}(5 b c-3 a d)\sqrt{1-\frac{b x^4}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{a^{1/4}}\right],-1\right]}{8 b^{1/4} c^2 (b c-a d)\sqrt{a-b x^4}}$$

Result (type 6, 349 leaves):

$$\left(x\left(\frac{5\,d\,(a-b\,x^4)}{c\,(b\,c-a\,d)}+\left(25\,a\,(-4\,b\,c+3\,a\,d)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/\left((b\,c-a\,d)\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)+\left(9\,a\,b\,d\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/\left((-b\,c+a\,d)\left(9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(20\,\sqrt{a-b\,x^4}\,(-c+d\,x^4)\right)$$

Problem 90: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a-b\,x^4)^{3/2}\,(c-d\,x^4)^2}\,dx$$

Optimal (type 4, 362 leaves, 10 steps):

$$\frac{b\,(2\,b\,c+a\,d)\,x}{4\,a\,c\,(b\,c-a\,d)^2\,\sqrt{a-b\,x^4}}-\frac{d\,x}{4\,c\,(b\,c-a\,d)\,\sqrt{a-b\,x^4}\,(c-d\,x^4)}+\frac{b^{3/4}\,(2\,b\,c+a\,d)\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]}{4\,a^{3/4}\,c\,(b\,c-a\,d)^2\,\sqrt{a-b\,x^4}}-\left(3\,a^{1/4}\,d\,(3\,b\,c-a\,d)\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]\right)\Big/\left(8\,b^{1/4}\,c^2\,(b\,c-a\,d)^2\,\sqrt{a-b\,x^4}\right)-\left(3\,a^{1/4}\,d\,(3\,b\,c-a\,d)\,\sqrt{1-\frac{b\,x^4}{a}}\,\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]\right)\Big/\left(8\,b^{1/4}\,c^2\,(b\,c-a\,d)^2\,\sqrt{a-b\,x^4}\right)$$

Result (type 6, 465 leaves):

$$\Bigg(x\,\Bigg(\bigg(25\,\left(2\,b^2\,c^2-8\,a\,b\,c\,d+3\,a^2\,d^2\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\bigg)\Big/$$

$$\bigg(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+$$

$$2\,x^4\,\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\bigg)+$$

$$\bigg(9\,a\,c\,\left(5\,a^2\,d^2-6\,a\,b\,d^2\,x^4+2\,b^2\,c\,\left(5\,c-6\,d\,x^4\right)\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]-$$

$$10\,x^4\,\left(-a^2\,d^2+a\,b\,d^2\,x^4-2\,b^2\,c\,\left(c-d\,x^4\right)\right)$$

$$\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\bigg)\Big/$$

$$\bigg(a\,c\,\bigg(9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+$$

$$2\,x^4\,\bigg(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+$$

$$b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\bigg)\bigg)\bigg)\Bigg)\Bigg)\Big/\left(20\,\left(b\,c-a\,d\right)^2\,\sqrt{a-b\,x^4}\,\left(c-d\,x^4\right)\right)$$

Problem 91: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a-b\,x^4\right)^{5/2}\,\left(c-d\,x^4\right)^2}\,dx$$

Optimal (type 4, 439 leaves, 11 steps):

$$\frac{b\,(2\,b\,c+3\,a\,d)\,x}{12\,a\,c\,(b\,c-a\,d)^2\,(a-b\,x^4)^{3/2}}+\frac{b\,(5\,b^2\,c^2-17\,a\,b\,c\,d-3\,a^2\,d^2)\,x}{12\,a^2\,c\,(b\,c-a\,d)^3\,\sqrt{a-b\,x^4}}-\frac{d\,x}{4\,c\,(b\,c-a\,d)\,(a-b\,x^4)^{3/2}\,(c-d\,x^4)}+$$

$$\left(b^{3/4}\,(5\,b^2\,c^2-17\,a\,b\,c\,d-3\,a^2\,d^2)\,\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]\right)\Big/$$

$$\left(12\,a^{7/4}\,c\,(b\,c-a\,d)^3\,\sqrt{a-b\,x^4}\right)+$$

$$\left(a^{1/4}\,d^2\,(13\,b\,c-3\,a\,d)\,\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticPi}\left[-\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]\right)\Big/$$

$$\left(8\,b^{1/4}\,c^2\,(b\,c-a\,d)^3\,\sqrt{a-b\,x^4}\right)+$$

$$\left(a^{1/4}\,d^2\,(13\,b\,c-3\,a\,d)\,\sqrt{1-\frac{b\,x^4}{a}}\;\text{EllipticPi}\left[\frac{\sqrt{a}\,\sqrt{d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{a^{1/4}}\right],-1\right]\right)\Big/$$

$$\left(8\,b^{1/4}\,c^2\,(b\,c-a\,d)^3\,\sqrt{a-b\,x^4}\right)$$

Result (type 6, 617 leaves):

$$\frac{1}{60\,a^2\,(-b\,c+a\,d)^3\,\sqrt{a-b\,x^4}\,(c-d\,x^4)}$$

$$x\left(\left(25\,a\,(-5\,b^3\,c^3+17\,a\,b^2\,c^2\,d-36\,a^2\,b\,c\,d^2+9\,a^3\,d^3)\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\Big/\right.$$

$$\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{1}{2},1,\frac{5}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+\right.$$

$$\left.2\,x^4\left(2\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},2,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)+$$

$$\left(9\,a\,c\,\left(15\,a^4\,d^3-33\,a^3\,b\,d^3\,x^4+5\,b^4\,c^2\,x^4\,(5\,c-6\,d\,x^4)+a^2\,b^2\,d\,(95\,c^2-112\,c\,d\,x^4+18\,d^2\,x^8)+\right.\right.$$

$$\left.a\,b^3\,c\,(-35\,c^2-45\,c\,d\,x^4+102\,d^2\,x^8)\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+$$

$$10\,x^4\,\left(3\,a^4\,d^3-6\,a^3\,b\,d^3\,x^4+5\,b^4\,c^2\,x^4\,(c-d\,x^4)+a^2\,b^2\,d\,(19\,c^2-19\,c\,d\,x^4+3\,d^2\,x^8)+\right.$$

$$\left.a\,b^3\,c\,(-7\,c^2-10\,c\,d\,x^4+17\,d^2\,x^8)\right)$$

$$\left.\left(2\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\Big/$$

$$\left(c\,(a-b\,x^4)\,\left(9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+2\,x^4\,\left(2\,a\,d\right.\right.\right.$$

$$\left.\left.\left.\left.\text{AppellF1}\left[\frac{9}{4},\frac{1}{2},2,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{2},1,\frac{13}{4},\frac{b\,x^4}{a},\frac{d\,x^4}{c}\right]\right)\right)\right)\right)$$

Problem 92: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+b x^4}}{a c-b c x^4} \, dx$$

Optimal (type 3, 103 leaves, 4 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{2} a^{1/4} b^{1/4} x}{\sqrt{a+b x^4}}\right]}{2 \sqrt{2} a^{1/4} b^{1/4} c}+\frac{\text{ArcTanh}\left[\frac{\sqrt{2} a^{1/4} b^{1/4} x}{\sqrt{a+b x^4}}\right]}{2 \sqrt{2} a^{1/4} b^{1/4} c}$$

Result (type 6, 155 leaves):

$$\left(5 a x \sqrt{a+b x^4} \text{ AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{b x^4}{a},\frac{b x^4}{a}\right]\right) \Big/ \left(c \left(a-b x^4\right) \left(5 a \text{ AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},-\frac{b x^4}{a},\frac{b x^4}{a}\right]+2 b x^4 \left(2 \text{ AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},-\frac{b x^4}{a},\frac{b x^4}{a}\right]+\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},-\frac{b x^4}{a},\frac{b x^4}{a}\right]\right)\right)\right)$$

Problem 93: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a-b x^4}}{a c+b c x^4} \, dx$$

Optimal (type 3, 116 leaves, 1 step):

$$\frac{\text{ArcTan}\left[\frac{b^{1/4} x \left(\sqrt{a}+\sqrt{b} x^2\right)}{a^{1/4} \sqrt{a-b x^4}}\right]}{2 a^{1/4} b^{1/4} c}+\frac{\text{ArcTanh}\left[\frac{b^{1/4} x \left(\sqrt{a}-\sqrt{b} x^2\right)}{a^{1/4} \sqrt{a-b x^4}}\right]}{2 a^{1/4} b^{1/4} c}$$

Result (type 6, 155 leaves):

$$\left(5 a x \sqrt{a-b x^4} \text{ AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},-\frac{b x^4}{a}\right]\right) \Big/ \left(c \left(a+b x^4\right) \left(5 a \text{ AppellF1}\left[\frac{1}{4},-\frac{1}{2},1,\frac{5}{4},\frac{b x^4}{a},-\frac{b x^4}{a}\right]-2 b x^4 \left(2 \text{ AppellF1}\left[\frac{5}{4},-\frac{1}{2},2,\frac{9}{4},\frac{b x^4}{a},-\frac{b x^4}{a}\right]+\text{AppellF1}\left[\frac{5}{4},\frac{1}{2},1,\frac{9}{4},\frac{b x^4}{a},-\frac{b x^4}{a}\right]\right)\right)\right)$$

Problem 94: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{7/4}}{c+d x^4} \, dx$$

Optimal (type 3, 211 leaves, 10 steps):

$$\frac{b\,x\,\left(a+b\,x^4\right)^{3/4}}{4\,d}-\frac{b^{3/4}\,\left(4\,b\,c-7\,a\,d\right)\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right]}{8\,d^2}+\frac{\left(b\,c-a\,d\right)^{7/4}\,\text{ArcTan}\left[\frac{\left(b\,c-a\,d\right)^{1/4}\,x}{c^{1/4}\,\left(a+b\,x^4\right)^{1/4}}\right]}{2\,c^{3/4}\,d^2}-\frac{b^{3/4}\,\left(4\,b\,c-7\,a\,d\right)\,\text{ArcTanh}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right]}{8\,d^2}+\frac{\left(b\,c-a\,d\right)^{7/4}\,\text{ArcTanh}\left[\frac{\left(b\,c-a\,d\right)^{1/4}\,x}{c^{1/4}\,\left(a+b\,x^4\right)^{1/4}}\right]}{2\,c^{3/4}\,d^2}$$

Result (type 6, 396 leaves):

$$\frac{1}{80}\left(-\left(\left(36\,a\,b\,c\,\left(-4\,b\,c+7\,a\,d\right)\,x^5\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(d\,\left(a+b\,x^4\right)^{1/4}\,\left(c+d\,x^4\right)\,\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\,\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{4},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{5}{4},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)+\left(5\,\left(4\,b\,c^{3/4}\,\left(b\,c-a\,d\right)^{1/4}\,x\,\left(a+b\,x^4\right)^{3/4}+2\,a\,\left(-b\,c+4\,a\,d\right)\,\text{ArcTan}\left[\frac{\left(b\,c-a\,d\right)^{1/4}\,x}{c^{1/4}\,\left(b+a\,x^4\right)^{1/4}}\right]+a\,\left(b\,c-4\,a\,d\right)\,\text{Log}\left[c^{1/4}-\frac{\left(b\,c-a\,d\right)^{1/4}\,x}{\left(b+a\,x^4\right)^{1/4}}\right]-a\,b\,c\,\text{Log}\left[c^{1/4}+\frac{\left(b\,c-a\,d\right)^{1/4}\,x}{\left(b+a\,x^4\right)^{1/4}}\right]+4\,a^2\,d\,\text{Log}\left[c^{1/4}+\frac{\left(b\,c-a\,d\right)^{1/4}\,x}{\left(b+a\,x^4\right)^{1/4}}\right]\right)\right)\Big/\left(c^{3/4}\,d\,\left(b\,c-a\,d\right)^{1/4}\right)\right)$$

Problem 95: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^4\right)^{3/4}}{c+d\,x^4}\,dx$$

Optimal (type 3, 173 leaves, 9 steps):

$$\frac{b^{3/4}\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right]}{2\,d}-\frac{\left(b\,c-a\,d\right)^{3/4}\,\text{ArcTan}\left[\frac{\left(b\,c-a\,d\right)^{1/4}\,x}{c^{1/4}\,\left(a+b\,x^4\right)^{1/4}}\right]}{2\,c^{3/4}\,d}+\frac{b^{3/4}\,\text{ArcTanh}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right]}{2\,d}-\frac{\left(b\,c-a\,d\right)^{3/4}\,\text{ArcTanh}\left[\frac{\left(b\,c-a\,d\right)^{1/4}\,x}{c^{1/4}\,\left(a+b\,x^4\right)^{1/4}}\right]}{2\,c^{3/4}\,d}$$

Result (type 6, 161 leaves):

$$\left(5\,a\,c\,x\,\left(a+b\,x^4\right)^{3/4}\,\text{AppellF1}\left[\frac{1}{4},-\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\,\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},-\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\,\left(-4\,a\,d\,\text{AppellF1}\left[\frac{5}{4},-\frac{3}{4},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)$$

Problem 100: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{9/4}}{c+d x^4}\, dx$$

Optimal (type 4, 316 leaves, 11 steps):

$$-\frac{b\,(6 b c-11 a d)\, x\,\left(a+b x^4\right)^{1/4}}{12 d^2}+\frac{b x\left(a+b x^4\right)^{5/4}}{6 d}+\left(\sqrt{a}\, b^{3/2}\,(6 b c-11 a d)\left(1+\frac{a}{b x^4}\right)^{3/4} x^3\, \text{EllipticF}\left[\frac{1}{2}\text{ArcCot}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]\right)\bigg/\left(12 d^2\left(a+b x^4\right)^{3/4}\right)+\frac{1}{2 b^{1/4} c\, d^2}(b c-a d)^2\sqrt{\frac{a}{a+b x^4}}\sqrt{a+b x^4}\,\text{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]+\frac{1}{2 b^{1/4} c\, d^2}(b c-a d)^2\sqrt{\frac{a}{a+b x^4}}\sqrt{a+b x^4}\,\text{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]$$

Result (type 6, 396 leaves):

$$\frac{1}{60 d^2\left(a+b x^4\right)^{3/4}} x\left(5 b\left(a+b x^4\right)\left(-6 b c+13 a d+2 b d x^4\right)-\left(25 a^2 c\left(6 b^2 c^2-13 a b c d+12 a^2 d^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\bigg/\left(\left(c+d x^4\right)\left(-5 a c\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4\left(4 a d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c\,\text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)-\left(9 a b c\left(12 b^2 c^2-30 a b c d+23 a^2 d^2\right) x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\bigg/\left(\left(c+d x^4\right)\left(-9 a c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4\left(4 a d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c\,\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)\right)$$

Problem 101: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{5/4}}{c+d x^4}\, dx$$

Optimal (type 4, 274 leaves, 10 steps):

$$\frac{b\,x\,\left(a+b\,x^4\right)^{1/4}}{2\,d}-\frac{\sqrt{a}\,b^{3/2}\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{2\,d\,\left(a+b\,x^4\right)^{3/4}}-\frac{1}{2\,b^{1/4}\,c\,d}\left(b\,c-a\,d\right)\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[-\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],\,-1\right]-\frac{1}{2\,b^{1/4}\,c\,d}\left(b\,c-a\,d\right)\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],\,-1\right]$$

Result (type 6, 435 leaves):

$$\left(x\left(-\left(\left(25\,a^2\,c\,\left(-b\,c+2\,a\,d\right)\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)+\left(b\left(-9\,a\,c\,\left(5\,a\,c+3\,b\,c\,x^4+8\,a\,d\,x^4+5\,b\,d\,x^8\right)\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+5\,x^4\,\left(a+b\,x^4\right)\,\left(c+d\,x^4\right)\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\Big/\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)\Big/\left(10\,d\,\left(a+b\,x^4\right)^{3/4}\,\left(c+d\,x^4\right)\right)$$

Problem 102: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^4\right)^{1/4}}{c+d\,x^4}\,dx$$

Optimal (type 4, 166 leaves, 4 steps):

$$\frac{\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[-\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],\,-1\right]}{2\,b^{1/4}\,c}+\frac{\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],\,-1\right]}{2\,b^{1/4}\,c}$$

Result (type 6, 160 leaves):

$$\left(5\,a\,c\,x\,\left(a+b\,x^4\right)^{1/4}\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},-\frac{1}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(-4\,a\,d\,\text{AppellF1}\left[\frac{5}{4},-\frac{1}{4},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)$$

Problem 103: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b\,x^4\right)^{3/4}\left(c+d\,x^4\right)}\,dx$$

Optimal (type 4, 259 leaves, 9 steps):

$$-\frac{b^{3/2}\left(1+\frac{a}{b\,x^4}\right)^{3/4}x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{\sqrt{a}\,\left(b\,c-a\,d\right)\left(a+b\,x^4\right)^{3/4}}-\frac{d\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[-\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,\left(b\,c-a\,d\right)}-\frac{d\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],-1\right]}{2\,b^{1/4}\,c\,\left(b\,c-a\,d\right)}$$

Result (type 6, 161 leaves):

$$-\left(\left(5\,a\,c\,x\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(a+b\,x^4\right)^{3/4}\left(c+d\,x^4\right)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)$$

Problem 104: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b\,x^4\right)^{7/4}\left(c+d\,x^4\right)}\,dx$$

Optimal (type 4, 304 leaves, 10 steps):

$$\frac{b\,x}{3\,a\,(b\,c-a\,d)\,(a+b\,x^4)^{3/4}}-\frac{b^{3/2}\,(2\,b\,c-5\,a\,d)\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{3\,a^{3/2}\,(b\,c-a\,d)^2\,(a+b\,x^4)^{3/4}}+$$

$$\frac{d^2\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[-\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{(a+b\,x^4)^{1/4}}\right],\,-1\right]}{2\,b^{1/4}\,c\,(b\,c-a\,d)^2}+$$

$$\frac{d^2\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{(a+b\,x^4)^{1/4}}\right],\,-1\right]}{2\,b^{1/4}\,c\,(b\,c-a\,d)^2}$$

Result (type 6, 343 leaves):

$$\left(x\left(-\frac{5\,b}{a}+\left(25\,c\,(2\,b\,c-3\,a\,d)\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\right.\right.$$
$$\left(\left(c+d\,x^4\right)\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)+$$
$$\left(18\,b\,c\,d\,x^4\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left(\left(c+d\,x^4\right)\left(-9\,a\,c\,\text{AppellF1}\left[\right.\right.\right.$$
$$\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+\right.$$
$$\left.\left.\left.\left.\left.3\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(15\,(-b\,c+a\,d)\,\left(a+b\,x^4\right)^{3/4}\right)$$

Problem 105: Result unnecessarily involves higher level functions.

$$\int\frac{1}{\left(a+b\,x^4\right)^{11/4}\,\left(c+d\,x^4\right)}\,dx$$

Optimal (type 4, 357 leaves, 11 steps):

$$\frac{b x}{7 a (b c-a d) (a+b x^4)^{7/4}}+\frac{b (6 b c-13 a d) x}{21 a^2 (b c-a d)^2 (a+b x^4)^{3/4}}-$$
$$\left(b^{3/2} (12 b^2 c^2-38 a b c d+47 a^2 d^2) \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]\right)\Big/$$
$$\left(21 a^{5/2} (b c-a d)^3 (a+b x^4)^{3/4}\right)-$$
$$\frac{d^3 \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4} \text{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{(a+b x^4)^{1/4}}\right],-1\right]}{2 b^{1/4} c (b c-a d)^3}-$$
$$\frac{d^3 \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4} \text{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{(a+b x^4)^{1/4}}\right],-1\right]}{2 b^{1/4} c (b c-a d)^3}$$

Result (type 6, 407 leaves):

$$\left(x \left(\frac{5 b (-16 a^2 d+6 b^2 c x^4+a b (9 c-13 d x^4))}{a+b x^4}+\right.\right.$$
$$\left(25 a c (12 b^2 c^2-26 a b c d+21 a^2 d^2) \text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/$$
$$\left((c+d x^4) \left(5 a c \text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]-x^4 \left(4 a d \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c \text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)+$$
$$\left(18 a b c d (-6 b c+13 a d) x^4 \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/$$
$$\left((c+d x^4) \left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+\right.\right.$$
$$x^4 \left(4 a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c\right.$$
$$\left.\left.\left.\left.\left.\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)\right)\right)\Big/\left(105 a^2 (b c-a d)^2 (a+b x^4)^{3/4}\right)$$

Problem 106: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a+b x^4)^{11/4}}{(c+d x^4)^2} \, dx$$

Optimal (type 3, 280 leaves, 11 steps):

$$\frac{b\,(2\,b\,c-a\,d)\,x\,(a+b\,x^4)^{3/4}}{4\,c\,d^2}-\frac{(b\,c-a\,d)\,x\,(a+b\,x^4)^{7/4}}{4\,c\,d\,(c+d\,x^4)}-$$
$$\frac{b^{7/4}\,(8\,b\,c-11\,a\,d)\,\text{ArcTan}\left[\frac{b^{1/4}\,x}{(a+b\,x^4)^{1/4}}\right]}{8\,d^3}+\frac{(b\,c-a\,d)^{7/4}\,(8\,b\,c+3\,a\,d)\,\text{ArcTan}\left[\frac{(b\,c-a\,d)^{1/4}\,x}{c^{1/4}\,(a+b\,x^4)^{1/4}}\right]}{8\,c^{7/4}\,d^3}-$$
$$\frac{b^{7/4}\,(8\,b\,c-11\,a\,d)\,\text{ArcTanh}\left[\frac{b^{1/4}\,x}{(a+b\,x^4)^{1/4}}\right]}{8\,d^3}+\frac{(b\,c-a\,d)^{7/4}\,(8\,b\,c+3\,a\,d)\,\text{ArcTanh}\left[\frac{(b\,c-a\,d)^{1/4}\,x}{c^{1/4}\,(a+b\,x^4)^{1/4}}\right]}{8\,c^{7/4}\,d^3}$$

Result (type 6, 735 leaves):

$$\frac{1}{80\,c^{7/4}\,d^2\,(c+d\,x^4)}$$
$$\left(-\left(\left(36\,a\,b^2\,c^{11/4}\,(-8\,b\,c+11\,a\,d)\,x^5\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\Big/\left((a+b\,x^4)^{1/4}\right.\right.\right.$$
$$\left(-9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+x^4\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\frac{1}{4},2,\frac{13}{4},\right.\right.\right.$$
$$\left.\left.\left.\left.\left.-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]+b\,c\,\text{AppellF1}\left[\frac{9}{4},\frac{5}{4},1,\frac{13}{4},-\frac{b\,x^4}{a},-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)+$$
$$\frac{1}{(b\,c-a\,d)^{1/4}}\,5\,\left(8\,b^2\,c^{11/4}\,(b\,c-a\,d)^{1/4}\,x\,(a+b\,x^4)^{3/4}-8\,a\,b\,c^{7/4}\,d\,(b\,c-a\,d)^{1/4}\,x\,(a+b\,x^4)^{3/4}+\right.$$
$$4\,a^2\,c^{3/4}\,d^2\,(b\,c-a\,d)^{1/4}\,x\,(a+b\,x^4)^{3/4}+4\,b^2\,c^{7/4}\,d\,(b\,c-a\,d)^{1/4}\,x^5\,(a+b\,x^4)^{3/4}+$$
$$2\,a\,(-2\,b^2\,c^2+2\,a\,b\,c\,d+3\,a^2\,d^2)\,(c+d\,x^4)\,\text{ArcTan}\left[\frac{(b\,c-a\,d)^{1/4}\,x}{c^{1/4}\,(b+a\,x^4)^{1/4}}\right]-$$
$$a\,(-2\,b^2\,c^2+2\,a\,b\,c\,d+3\,a^2\,d^2)\,(c+d\,x^4)\,\text{Log}\left[c^{1/4}-\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]-$$
$$2\,a\,b^2\,c^3\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]+2\,a^2\,b\,c^2\,d\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]+$$
$$3\,a^3\,c\,d^2\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]-2\,a\,b^2\,c^2\,d\,x^4\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]+$$
$$\left.\left.2\,a^2\,b\,c\,d^2\,x^4\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]+3\,a^3\,d^3\,x^4\,\text{Log}\left[c^{1/4}+\frac{(b\,c-a\,d)^{1/4}\,x}{(b+a\,x^4)^{1/4}}\right]\right)\right)$$

Problem 107: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{(a+b\,x^4)^{7/4}}{(c+d\,x^4)^2}\,dx$$

Optimal (type 3, 230 leaves, 10 steps):

$$-\frac{(b c-a d)\, x\, \left(a+b x^4\right)^{3/4}}{4 c d \left(c+d x^4\right)}+\frac{b^{7/4} \operatorname{ArcTan}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{2 d^2}-\frac{(b c-a d)^{3/4} (4 b c+3 a d) \operatorname{ArcTan}\left[\frac{(b c-a d)^{1/4} x}{c^{1/4} \left(a+b x^4\right)^{1/4}}\right]}{8 c^{7/4} d^2}+\frac{b^{7/4} \operatorname{ArcTanh}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right]}{2 d^2}-\frac{(b c-a d)^{3/4} (4 b c+3 a d) \operatorname{ArcTanh}\left[\frac{(b c-a d)^{1/4} x}{c^{1/4} \left(a+b x^4\right)^{1/4}}\right]}{8 c^{7/4} d^2}$$

Result (type 6, 462 leaves):

$$-\frac{(b c-a d)\, x\, \left(a+b x^4\right)^{3/4}}{4 c d \left(c+d x^4\right)}-\left(9 a b^2 c x^5 \operatorname{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(5 d \left(a+b x^4\right)^{1/4} \left(c+d x^4\right)\left(-9 a c \operatorname{AppellF1}\left[\frac{5}{4},\frac{1}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4 \left(4 a d \operatorname{AppellF1}\left[\frac{9}{4},\frac{1}{4},2,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+b c \operatorname{AppellF1}\left[\frac{9}{4},\frac{5}{4},1,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)+\left(3 a^2 \left(2 \operatorname{ArcTan}\left[\frac{(b c-a d)^{1/4} x}{c^{1/4} \left(b+a x^4\right)^{1/4}}\right]-\operatorname{Log}\left[c^{1/4}-\frac{(b c-a d)^{1/4} x}{\left(b+a x^4\right)^{1/4}}\right]+\operatorname{Log}\left[c^{1/4}+\frac{(b c-a d)^{1/4} x}{\left(b+a x^4\right)^{1/4}}\right]\right)\right)\Big/\left(16 c^{7/4} (b c-a d)^{1/4}\right)+\left(a b \left(2 \operatorname{ArcTan}\left[\frac{(b c-a d)^{1/4} x}{c^{1/4} \left(b+a x^4\right)^{1/4}}\right]-\operatorname{Log}\left[c^{1/4}-\frac{(b c-a d)^{1/4} x}{\left(b+a x^4\right)^{1/4}}\right]+\operatorname{Log}\left[c^{1/4}+\frac{(b c-a d)^{1/4} x}{\left(b+a x^4\right)^{1/4}}\right]\right)\right)\Big/\left(16 c^{3/4} d (b c-a d)^{1/4}\right)$$

Problem 112: Result unnecessarily involves higher level functions.

$$\int \frac{\left(a+b x^4\right)^{9/4}}{\left(c+d x^4\right)^2} \, dx$$

Optimal (type 4, 353 leaves, 11 steps):

$$\frac{b\left(3bc-ad\right)x\left(a+bx^4\right)^{1/4}}{4cd^2}-\frac{\left(bc-ad\right)x\left(a+bx^4\right)^{5/4}}{4cd\left(c+dx^4\right)}-$$
$$\frac{\sqrt{a}\,b^{3/2}\left(3bc-ad\right)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left[\frac{1}{2}\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],2\right]}{4cd^2\left(a+bx^4\right)^{3/4}}-$$
$$\frac{1}{8b^{1/4}c^2d^2}3\left(bc-ad\right)\left(2bc+ad\right)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}$$
$$\text{EllipticPi}\left[-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}x}{\left(a+bx^4\right)^{1/4}}\right],-1\right]-\frac{1}{8b^{1/4}c^2d^2}$$
$$3\left(bc-ad\right)\left(2bc+ad\right)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\text{EllipticPi}\left[\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4}x}{\left(a+bx^4\right)^{1/4}}\right],-1\right]$$

Result (type 6, 506 leaves):

$$\Bigg(x\Bigg(-\Bigg(\Bigg(25a^2\left(-3b^2c^2+2abcd+3a^2d^2\right)\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\Bigg)\Bigg/$$
$$\Bigg(-5ac\,\text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+x^4\Bigg(4ad\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\right.$$
$$\left.\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+3bc\,\text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\Bigg)\Bigg)\Bigg)+$$
$$\Bigg(-9ac\left(5a^3d^2+3ab^2c\left(5c+2dx^4\right)+a^2bd\left(-10c+7dx^4\right)+b^3cx^4\left(9c+10dx^4\right)\right)$$
$$\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+5x^4\left(a+bx^4\right)$$
$$\left(-2abcd+a^2d^2+b^2c\left(3c+2dx^4\right)\right)\Bigg(4ad\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+$$
$$3bc\,\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\Bigg)\Bigg)\Bigg/$$
$$\Bigg(c\Bigg(-9ac\,\text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+$$
$$x^4\Bigg(4ad\,\text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]+$$
$$3bc\,\text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{bx^4}{a},-\frac{dx^4}{c}\right]\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg/\left(20d^2\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)\right)$$

Problem 113: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+bx^4\right)^{5/4}}{\left(c+dx^4\right)^2}\,dx$$

Optimal (type 4, 298 leaves, 10 steps):

$$-\frac{\left(b\,c-a\,d\right)\,x\,\left(a+b\,x^4\right)^{1/4}}{4\,c\,d\,\left(c+d\,x^4\right)}+\frac{\sqrt{a}\,b^{3/2}\,\left(1+\frac{a}{b\,x^4}\right)^{3/4}\,x^3\,\text{EllipticF}\left[\frac{1}{2}\,\text{ArcCot}\left[\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right],\,2\right]}{4\,c\,d\,\left(a+b\,x^4\right)^{3/4}}+\frac{1}{8\,b^{1/4}\,c^2\,d}$$

$$\left(2\,b\,c+3\,a\,d\right)\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[-\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],\,-1\right]+$$

$$\frac{1}{8\,b^{1/4}\,c^2\,d}\left(2\,b\,c+3\,a\,d\right)\,\sqrt{\frac{a}{a+b\,x^4}}\,\sqrt{a+b\,x^4}\,\text{EllipticPi}\left[\frac{\sqrt{b\,c-a\,d}}{\sqrt{b}\,\sqrt{c}},\,\text{ArcSin}\left[\frac{b^{1/4}\,x}{\left(a+b\,x^4\right)^{1/4}}\right],\,-1\right]$$

Result (type 6, 440 leaves) :

$$\left(x\,\left(-\left(\left(25\,a^2\,\left(b\,c+3\,a\,d\right)\,\text{AppellF1}\left[\frac{1}{4},\,\frac{3}{4},\,1,\,\frac{5}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]\right)\right/\right.\right.$$

$$\left(-5\,a\,c\,\text{AppellF1}\left[\frac{1}{4},\,\frac{3}{4},\,1,\,\frac{5}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]+x^4\,\left(4\,a\,d\,\text{AppellF1}\left[\frac{5}{4},\,\frac{3}{4},\,2,\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{9}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]+3\,b\,c\,\text{AppellF1}\left[\frac{5}{4},\,\frac{7}{4},\,1,\,\frac{9}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]\right)\right)\right)+$$

$$\left(9\,a\,c\,\left(5\,a^2\,d-3\,b^2\,c\,x^4+a\,b\,\left(-5\,c+7\,d\,x^4\right)\right)\,\text{AppellF1}\left[\frac{5}{4},\,\frac{3}{4},\,1,\,\frac{9}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]+\right.$$

$$5\,\left(b\,c-a\,d\right)\,x^4\,\left(a+b\,x^4\right)\,\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\,\frac{3}{4},\,2,\,\frac{13}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]+\right.$$

$$\left.\left.3\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\,\frac{7}{4},\,1,\,\frac{13}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]\right)\right)\right/$$

$$\left(c\,\left(9\,a\,c\,\text{AppellF1}\left[\frac{5}{4},\,\frac{3}{4},\,1,\,\frac{9}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]-\right.\right.$$

$$x^4\,\left(4\,a\,d\,\text{AppellF1}\left[\frac{9}{4},\,\frac{3}{4},\,2,\,\frac{13}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]+\right.$$

$$\left.\left.\left.\left.\left.3\,b\,c\,\text{AppellF1}\left[\frac{9}{4},\,\frac{7}{4},\,1,\,\frac{13}{4},\,-\frac{b\,x^4}{a},\,-\frac{d\,x^4}{c}\right]\right)\right)\right)\right)\right)\right/\left(20\,d\,\left(a+b\,x^4\right)^{3/4}\,\left(c+d\,x^4\right)\right)$$

Problem 114: Result unnecessarily involves higher level functions.

$$\int\frac{\left(a+b\,x^4\right)^{1/4}}{\left(c+d\,x^4\right)^2}\,dx$$

Optimal (type 4, 308 leaves, 10 steps) :

$$\frac{x\left(a+b x^4\right)^{1/4}}{4 c\left(c+d x^4\right)}-\frac{\sqrt{a}\, b^{3/2}\left(1+\frac{a}{b x^4}\right)^{3/4} x^3\, \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{4 c (b c-a d)\left(a+b x^4\right)^{3/4}}+\left((2 b c-3 a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4}\, \text{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]\right)\Big/\left(8 b^{1/4} c^2 (b c-a d)\right)+\left((2 b c-3 a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4}\, \text{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]\right)\Big/\left(8 b^{1/4} c^2 (b c-a d)\right)$$

Result (type 6, 322 leaves):

$$\left(x\left(\frac{5\left(a+b x^4\right)}{c}-\left(75 a^2 \text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(-5 a c\, \text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4\left(4 a d\, \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c\, \text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)-\left(18 a b x^4 \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(-9 a c\, \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4\left(4 a d\, \text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c\, \text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)\right)\Big/\left(20\left(a+b x^4\right)^{3/4}\left(c+d x^4\right)\right)$$

Problem 115: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^4\right)^{3/4}\left(c+d x^4\right)^2}\, dx$$

Optimal (type 4, 330 leaves, 10 steps):

$$-\frac{d x\left(a+b x^4\right)^{1/4}}{4 c (b c-a d)\left(c+d x^4\right)}-\frac{b^{3/2}(4 b c-a d)\left(1+\frac{a}{b x^4}\right)^{3/4} x^3\, \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b}\, x^2}{\sqrt{a}}\right],2\right]}{4 \sqrt{a}\, c (b c-a d)^2\left(a+b x^4\right)^{3/4}}-\left(3 d (2 b c-a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4}\, \text{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]\right)\Big/\left(8 b^{1/4} c^2 (b c-a d)^2\right)-\left(3 d (2 b c-a d) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4}\, \text{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}}, \text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]\right)\Big/\left(8 b^{1/4} c^2 (b c-a d)^2\right)$$

Result (type 6, 341 leaves):

$$\left(x \left(-\frac{5 d \left(a+b x^4\right)}{c}+\left(25 a \left(-4 b c+3 a d\right) \text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(-5 a c \text{AppellF1}\left[\frac{1}{4},\frac{3}{4},1,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4 \left(4 a d \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},2,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c \text{AppellF1}\left[\frac{5}{4},\frac{7}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)+\left(18 a b d x^4 \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/\left(-9 a c \text{AppellF1}\left[\frac{5}{4},\frac{3}{4},1,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+x^4 \left(4 a d \text{AppellF1}\left[\frac{9}{4},\frac{3}{4},2,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+3 b c \text{AppellF1}\left[\frac{9}{4},\frac{7}{4},1,\frac{13}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)\right)\Big/\left(20 \left(b c-a d\right) \left(a+b x^4\right)^{3/4} \left(c+d x^4\right)\right)$$

Problem 116: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\left(a+b x^4\right)^{7/4} \left(c+d x^4\right)^2} \, dx$$

Optimal (type 4, 390 leaves, 11 steps):

$$\frac{b \left(4 b c+3 a d\right) x}{12 a c \left(b c-a d\right)^2 \left(a+b x^4\right)^{3/4}}-\frac{d x}{4 c \left(b c-a d\right) \left(a+b x^4\right)^{3/4} \left(c+d x^4\right)}-\left(b^{3/2} \left(8 b^2 c^2-32 a b c d+3 a^2 d^2\right) \left(1+\frac{a}{b x^4}\right)^{3/4} x^3 \text{EllipticF}\left[\frac{1}{2} \text{ArcCot}\left[\frac{\sqrt{b} x^2}{\sqrt{a}}\right],2\right]\right)\Big/\left(12 a^{3/2} c \left(b c-a d\right)^3 \left(a+b x^4\right)^{3/4}\right)+\left(d^2 \left(10 b c-3 a d\right) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4} \text{EllipticPi}\left[-\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]\right)\Big/\left(8 b^{1/4} c^2 \left(b c-a d\right)^3\right)+\left(d^2 \left(10 b c-3 a d\right) \sqrt{\frac{a}{a+b x^4}} \sqrt{a+b x^4} \text{EllipticPi}\left[\frac{\sqrt{b c-a d}}{\sqrt{b} \sqrt{c}},\text{ArcSin}\left[\frac{b^{1/4} x}{\left(a+b x^4\right)^{1/4}}\right],-1\right]\right)\Big/\left(8 b^{1/4} c^2 \left(b c-a d\right)^3\right)$$

Result (type 6, 485 leaves):

$$\left(x \left(-\left(\left(25 \left(8 b^2 c^2 - 24 a b c d + 9 a^2 d^2\right) \text{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right) \Big/ \left(-5 a c \text{AppellF1}\left[\frac{1}{4}, \frac{3}{4}, 1, \frac{5}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right] + x^4 \left(4 a d \text{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 2, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right] + 3 b c \text{AppellF1}\left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\right)\right) + \left(9 a c \left(15 a^2 d^2 + 21 a b d^2 x^4 + 4 b^2 c \left(5 c + 7 d x^4\right)\right) \text{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right] - 5 x^4 \left(3 a^2 d^2 + 3 a b d^2 x^4 + 4 b^2 c \left(c + d x^4\right)\right) \left(4 a d \text{AppellF1}\left[\frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right] + 3 b c \text{AppellF1}\left[\frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\right) \Big/ \left(a c \left(9 a c \text{AppellF1}\left[\frac{5}{4}, \frac{3}{4}, 1, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right] - x^4 \left(4 a d \text{AppellF1}\left[\frac{9}{4}, \frac{3}{4}, 2, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right] + 3 b c \text{AppellF1}\left[\frac{9}{4}, \frac{7}{4}, 1, \frac{13}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\right)\right)\right)\right) \Big/ \left(60 \left(b c - a d\right)^2 \left(a + b x^4\right)^{3/4} \left(c + d x^4\right)\right)$$

Problem 119: Result more than twice size of optimal antiderivative.

$$\int \left(a + b x^4\right)^p \left(c + d x^4\right)^q \, dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$x \left(a + b x^4\right)^p \left(1 + \frac{b x^4}{a}\right)^{-p} \left(c + d x^4\right)^q \left(1 + \frac{d x^4}{c}\right)^{-q} \text{AppellF1}\left[\frac{1}{4}, -p, -q, \frac{5}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]$$

Result (type 6, 172 leaves):

$$\left(5 a c x \left(a + b x^4\right)^p \left(c + d x^4\right)^q \text{AppellF1}\left[\frac{1}{4}, -p, -q, \frac{5}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right) \Big/ \left(5 a c \text{AppellF1}\left[\frac{1}{4}, -p, -q, \frac{5}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right] + 4 x^4 \left(b c p \text{AppellF1}\left[\frac{5}{4}, 1 - p, -q, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right] + a d q \text{AppellF1}\left[\frac{5}{4}, -p, 1 - q, \frac{9}{4}, -\frac{b x^4}{a}, -\frac{d x^4}{c}\right]\right)\right)$$

Problem 122: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c + d x^4\right)^q}{a + b x^4} \, dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x\left(c+d x^4\right)^q\left(1+\frac{d x^4}{c}\right)^{-q}\text{AppellF1}\left[\frac{1}{4},1,-q,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]}{a}$$

Result (type 6, 162 leaves):

$$\left(5 a c x \left(c+d x^4\right)^q \text{AppellF1}\left[\frac{1}{4},-q,1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\Big/ \left(\left(a+b x^4\right)\left(5 a c\,\text{AppellF1}\left[\frac{1}{4},-q,1,\frac{5}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]+4 x^4\left(a d q\,\text{AppellF1}\left[\frac{5}{4},1-q,1,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]-b c\,\text{AppellF1}\left[\frac{5}{4},-q,2,\frac{9}{4},-\frac{d x^4}{c},-\frac{b x^4}{a}\right]\right)\right)\right)$$

Problem 123: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^4\right)^q}{\left(a+b x^4\right)^2}\,dx$$

Optimal (type 6, 57 leaves, 2 steps):

$$\frac{x\left(c+d x^4\right)^q\left(1+\frac{d x^4}{c}\right)^{-q}\text{AppellF1}\left[\frac{1}{4},2,-q,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]}{a^2}$$

Result (type 6, 162 leaves):

$$\left(5 a c x \left(c+d x^4\right)^q \text{AppellF1}\left[\frac{1}{4},2,-q,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\Big/ \left(\left(a+b x^4\right)^2\left(5 a c\,\text{AppellF1}\left[\frac{1}{4},2,-q,\frac{5}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]+4 x^4\left(a d q\,\text{AppellF1}\left[\frac{5}{4},2,1-q,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]-2 b c\,\text{AppellF1}\left[\frac{5}{4},3,-q,\frac{9}{4},-\frac{b x^4}{a},-\frac{d x^4}{c}\right]\right)\right)\right)$$

Problem 130: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

Optimal (type 3, 147 leaves, 8 steps):

$$\frac{2 d \sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)}+\frac{\sqrt{d}\,(3 b c-4 a d)\,\text{ArcTan}\left[\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{c^3\sqrt{b c-a d}}+\frac{(b c-4 a d)\,\text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{\sqrt{a}\,c^3}$$

Result (type 3, 197 leaves):

$$\frac{1}{2 c^3}\left(\frac{2 c \sqrt{a+\frac{b}{x}}\, x\, (2 d+c x)}{d+c x}+\frac{(b c-4 a d)\, \text{Log}\left[b+2 a x+2 \sqrt{a} \sqrt{a+\frac{b}{x}}\, x\right]}{\sqrt{a}}+\frac{i \sqrt{d}\, (3 b c-4 a d)\, \text{Log}\left[-\frac{2 i c^4 \left(-b d+b c x-2 a d x-2 i \sqrt{d} \sqrt{b c-a d} \sqrt{a+\frac{b}{x}}\, x\right)}{d^{3/2} (3 b c-4 a d) \sqrt{b c-a d}\, (d+c x)}\right]}{\sqrt{b c-a d}}\right)$$

Problem 131: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\, dx$$

Optimal (type 3, 213 leaves, 9 steps):

$$\frac{3 d \sqrt{a+\frac{b}{x}}}{2 c^2 \left(c+\frac{d}{x}\right)^2}+\frac{d (11 b c-12 a d) \sqrt{a+\frac{b}{x}}}{4 c^3 (b c-a d) \left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\, x}{c \left(c+\frac{d}{x}\right)^2}+\frac{\sqrt{d} \left(15 b^2 c^2-40 a b c d+24 a^2 d^2\right) \text{ArcTan}\left[\frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{4 c^4 (b c-a d)^{3/2}}+\frac{(b c-6 a d) \text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{\sqrt{a}\, c^4}$$

Result (type 3, 275 leaves):

$$\frac{1}{8\,c^4}\left(\left(2\,c\,\sqrt{a+\frac{b}{x}}\,x\,\left(-2\,a\,d\,\left(6\,d^2+9\,c\,d\,x+2\,c^2\,x^2\right)+b\,c\,\left(11\,d^2+17\,c\,d\,x+4\,c^2\,x^2\right)\right)\right)\Big/\left(\left(b\,c-a\,d\right)\,\left(d+c\,x\right)^2\right)+\frac{4\,\left(b\,c-6\,a\,d\right)\,\text{Log}\left[b+2\,a\,x+2\,\sqrt{a}\,\sqrt{a+\frac{b}{x}}\,x\right]}{\sqrt{a}}+\frac{1}{\left(b\,c-a\,d\right)^{3/2}}\,\mathrm{i}\,\sqrt{d}\,\left(15\,b^2\,c^2-40\,a\,b\,c\,d+24\,a^2\,d^2\right)\,\text{Log}\left[-\left(\left(8\,\mathrm{i}\,c^5\,\sqrt{b\,c-a\,d}\,\left(-b\,d+b\,c\,x-2\,a\,d\,x-2\,\mathrm{i}\,\sqrt{d}\,\sqrt{b\,c-a\,d}\,\sqrt{a+\frac{b}{x}}\,x\right)\right)\Big/\left(d^{3/2}\,\left(15\,b^2\,c^2-40\,a\,b\,c\,d+24\,a^2\,d^2\right)\,\left(d+c\,x\right)\right)\right)\right]\right)$$

Problem 137: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

Optimal (type 3, 156 leaves, 8 steps):

$$-\frac{\left(b\,c-2\,a\,d\right)\,\sqrt{a+\frac{b}{x}}}{c^2\,\left(c+\frac{d}{x}\right)}+\frac{a\,\sqrt{a+\frac{b}{x}}\,x}{c\,\left(c+\frac{d}{x}\right)}-\frac{\left(b\,c-4\,a\,d\right)\,\sqrt{b\,c-a\,d}\,\text{ArcTan}\left[\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{b\,c-a\,d}}\right]}{c^3\,\sqrt{d}}+\frac{\sqrt{a}\,\left(3\,b\,c-4\,a\,d\right)\,\text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{c^3}$$

Result (type 3, 231 leaves):

$$-\frac{1}{2c^3}\left(-\frac{2c\sqrt{a+\frac{b}{x}}\,x\,(-bc+2ad+acx)}{d+cx}+\sqrt{a}\,(-3bc+4ad)\,\text{Log}\left[b+2ax+2\sqrt{a}\sqrt{a+\frac{b}{x}}\,x\right]+\frac{1}{\sqrt{d}\sqrt{bc-ad}}\,\text{i}\,(b^2c^2-5abcd+4a^2d^2)\,\text{Log}\left[\frac{2c^4\left(-2\,\text{i}\,adx+2\sqrt{d}\sqrt{bc-ad}\sqrt{a+\frac{b}{x}}\,x-\text{i}\,b\,(d-cx)\right)}{\sqrt{d}\sqrt{bc-ad}\,(b^2c^2-5abcd+4a^2d^2)\,(d+cx)}\right]\right)$$

Problem 138: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

Optimal (type 3, 209 leaves, 9 steps):

$$-\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}-\frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)}+\frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2}-\frac{3(b^2c^2-8abcd+8a^2d^2)\,\text{ArcTan}\left[\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right]}{4c^4\sqrt{d}\sqrt{bc-ad}}+\frac{3\sqrt{a}\,(bc-2ad)\,\text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{c^4}$$

Result (type 3, 256 leaves):

$$\frac{1}{8c^4}\left(\frac{2c\sqrt{a+\frac{b}{x}}\,x\,\left(-bc\,(3d+5cx)+2a\,(6d^2+9cdx+2c^2x^2)\right)}{(d+cx)^2}-12\sqrt{a}\,(-bc+2ad)\,\text{Log}\left[b+2ax+2\sqrt{a}\sqrt{a+\frac{b}{x}}\,x\right]+\frac{1}{\sqrt{d}\sqrt{bc-ad}}\,3\,\text{i}\,(b^2c^2-8abcd+8a^2d^2)\,\text{Log}\left[\frac{8c^5\left(2\,\text{i}\,adx+2\sqrt{d}\sqrt{bc-ad}\sqrt{a+\frac{b}{x}}\,x+\text{i}\,b\,(d-cx)\right)}{3\sqrt{d}\sqrt{bc-ad}\,(b^2c^2-8abcd+8a^2d^2)\,(d+cx)}\right]\right)$$

Problem 144: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

Optimal (type 3, 166 leaves, 8 steps):

$$\frac{(b c-2 a d)(b c-a d)\sqrt{a+\frac{b}{x}}}{c^2 d\left(c+\frac{d}{x}\right)}+\frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)}-\frac{(b c-a d)^{3/2}(b c+4 a d)\,\text{ArcTan}\left[\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{c^3 d^{3/2}}+\frac{a^{3/2}(5 b c-4 a d)\,\text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{c^3}$$

Result (type 3, 219 leaves):

$$-\frac{1}{2 c^3}\left(-\frac{2 c\sqrt{a+\frac{b}{x}}\, x\left(b^2 c^2-2 a b c d+a^2 d(2 d+c x)\right)}{d(d+c x)}+a^{3/2}(-5 b c+4 a d)\,\text{Log}\left[b+2 a x+2\sqrt{a}\sqrt{a+\frac{b}{x}}\, x\right]+\frac{1}{d^{3/2}}\, i\,(b c-a d)^{3/2}(b c+4 a d)\,\text{Log}\left[\frac{2 c^4\left(-2 i a d^{3/2} x+2 d\sqrt{b c-a d}\sqrt{a+\frac{b}{x}}\, x-i b\sqrt{d}(d-c x)\right)}{(b c-a d)^{5/2}(b c+4 a d)(d+c x)}\right]\right)$$

Problem 145: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$$

Optimal (type 3, 237 leaves, 9 steps):

$$\frac{(b c-3 a d)(b c-a d)\sqrt{a+\frac{b}{x}}}{2 c^2 d\left(c+\frac{d}{x}\right)^2}-\frac{\left(b^2 c^2+7 a b c d-12 a^2 d^2\right)\sqrt{a+\frac{b}{x}}}{4 c^3 d\left(c+\frac{d}{x}\right)}+\frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2}-$$

$$\frac{\sqrt{b c-a d}\left(b^2 c^2+8 a b c d-24 a^2 d^2\right) \text{ArcTan}\left[\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{4 c^4 d^{3/2}}+\frac{a^{3/2}(5 b c-6 a d) \text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{c^4}$$

Result (type 3, 304 leaves):

$$\frac{1}{8 c^4}\left(\frac{1}{d (d+c x)^2} 2 c\sqrt{a+\frac{b}{x}}\, x\left(b^2 c^2(-d+c x)-a b c d(7 d+11 c x)+2 a^2 d\left(6 d^2+9 c d x+2 c^2 x^2\right)\right)-\right.$$

$$4 a^{3/2}(-5 b c+6 a d) \text{Log}\left[b+2 a x+2\sqrt{a}\sqrt{a+\frac{b}{x}}\, x\right]-$$

$$\frac{1}{d^{3/2}\sqrt{b c-a d}} i\left(b^3 c^3+7 a b^2 c^2 d-32 a^2 b c d^2+24 a^3 d^3\right)$$

$$\left.\text{Log}\left[\frac{8 c^5\left(-2 i a d^{3/2} x+2 d\sqrt{b c-a d}\sqrt{a+\frac{b}{x}}\, x-i b\sqrt{d}(d-c x)\right)}{\sqrt{b c-a d}\left(b^3 c^3+7 a b^2 c^2 d-32 a^2 b c d^2+24 a^3 d^3\right)(d+c x)}\right]\right)$$

Problem 151: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\, dx$$

Optimal (type 3, 172 leaves, 8 steps):

$$\frac{d(b c-2 a d)\sqrt{a+\frac{b}{x}}}{a c^2(b c-a d)\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\, x}{a c\left(c+\frac{d}{x}\right)}-$$

$$\frac{d^{3/2}(5 b c-4 a d) \text{ArcTan}\left[\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{c^3(b c-a d)^{3/2}}-\frac{(b c+4 a d) \text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{3/2} c^3}$$

Result (type 3, 224 leaves):

$$-\frac{1}{2c^3}\left(\frac{2c\sqrt{a+\frac{b}{x}}\,x\left(bc\left(d+cx\right)-ad\left(2d+cx\right)\right)}{a\left(-bc+ad\right)\left(d+cx\right)}+\right.$$

$$\frac{\left(bc+4ad\right)\mathrm{Log}\left[b+2ax+2\sqrt{a}\sqrt{a+\frac{b}{x}}\,x\right]}{a^{3/2}}+\frac{1}{\left(bc-ad\right)^{3/2}}\mathrm{i}\,d^{3/2}\left(5bc-4ad\right)$$

$$\mathrm{Log}\left[\left(2c^4\sqrt{bc-ad}\left(-2\,\mathrm{i}\,adx+2\sqrt{d}\sqrt{bc-ad}\sqrt{a+\frac{b}{x}}\,x-\mathrm{i}\,b\left(d-cx\right)\right)\right)\Big/\right.$$

$$\left.\left.\left(d^{5/2}\left(5bc-4ad\right)\left(d+cx\right)\right)\right]\right)$$

Problem 152: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$

Optimal (type 3, 250 leaves, 9 steps):

$$\frac{d\left(2bc-3ad\right)\sqrt{a+\frac{b}{x}}}{2ac^2\left(bc-ad\right)\left(c+\frac{d}{x}\right)^2}+\frac{d\left(bc-4ad\right)\left(4bc-3ad\right)\sqrt{a+\frac{b}{x}}}{4ac^3\left(bc-ad\right)^2\left(c+\frac{d}{x}\right)}+\frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2}-$$

$$\frac{d^{3/2}\left(35b^2c^2-56abcd+24a^2d^2\right)\mathrm{ArcTan}\left[\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right]}{4c^4\left(bc-ad\right)^{5/2}}-\frac{\left(bc+6ad\right)\mathrm{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{3/2}c^4}$$

Result (type 3, 301 leaves):

$$\frac{1}{8 c^4}\left(\left(2 c \sqrt{a+\frac{b}{x}}\, x \left(4 b^2 c^2 (d+c x)^2+2 a^2 d^2 \left(6 d^2+9 c d x+2 c^2 x^2\right)-a b c d \left(19 d^2+29 c d x+8 c^2 x^2\right)\right)\right)\Big/\left(a (b c-a d)^2 (d+c x)^2\right)-\frac{4 (b c+6 a d) \operatorname{Log}\left[b+2 a x+2 \sqrt{a} \sqrt{a+\frac{b}{x}}\, x\right]}{a^{3/2}}-\frac{1}{(b c-a d)^{5/2}} i\, d^{3/2} \left(35 b^2 c^2-56 a b c d+24 a^2 d^2\right) \operatorname{Log}\left[\left(8 c^5 (b c-a d)^{3/2} \left(-2 i a d x+2 \sqrt{d} \sqrt{b c-a d} \sqrt{a+\frac{b}{x}}\, x-i b (d-c x)\right)\right)\Big/\left(d^{5/2} \left(35 b^2 c^2-56 a b c d+24 a^2 d^2\right) (d+c x)\right)\right]\right)$$

Problem 158: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2}\, dx$$

Optimal (type 3, 224 leaves, 9 steps):

$$\frac{b \left(3 b^2 c^2-2 a b c d+2 a^2 d^2\right)}{a^2 c^2 (b c-a d)^2 \sqrt{a+\frac{b}{x}}}+\frac{d (b c-2 a d)}{a c^2 (b c-a d) \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)}+\frac{x}{a c \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)}+\frac{d^{5/2} (7 b c-4 a d) \operatorname{ArcTan}\left[\frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{c^3 (b c-a d)^{5/2}}-\frac{(3 b c+4 a d) \operatorname{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{5/2} c^3}$$

Result (type 3, 290 leaves):

$$\frac{1}{2 c^3}\left(\left(2 c \sqrt{a+\frac{b}{x}}\, x \left(3 b^3 c^2 (d+c x)+a^3 d^2 x (2 d+c x)+a^2 b d \left(2 d^2-c d x-2 c^2 x^2\right)+a b^2 c \left(-2 d^2-c d x+c^2 x^2\right)\right)\right) \Big/ \left(a^2 (b c-a d)^2 (b+a x) (d+c x)\right)-\frac{(3 b c+4 a d) \operatorname{Log}\left[b+2 a x+2 \sqrt{a} \sqrt{a+\frac{b}{x}}\, x\right]}{a^{5/2}}+\frac{1}{(b c-a d)^{5/2}} i\, d^{5/2} (7 b c-4 a d) \operatorname{Log}\left[-\left(\left(2 i c^4 (b c-a d)^{3/2} \left(-b d+b c x-2 a d x-2 i \sqrt{d} \sqrt{b c-a d} \sqrt{a+\frac{b}{x}}\, x\right)\right) \Big/ \left(d^{7/2} (7 b c-4 a d) (d+c x)\right)\right)\right]\right)$$

Problem 159: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3}\, dx$$

Optimal (type 3, 320 leaves, 10 steps):

$$\frac{3 b (2 b c-a d) \left(2 b^2 c^2-a b c d+4 a^2 d^2\right)}{4 a^2 c^3 (b c-a d)^3 \sqrt{a+\frac{b}{x}}}+\frac{d (2 b c-3 a d)}{2 a c^2 (b c-a d) \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^2}+\frac{d \left(4 b^2 c^2-21 a b c d+12 a^2 d^2\right)}{4 a c^3 (b c-a d)^2 \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)}+\frac{x}{a c \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^2}+\frac{3 d^{5/2} \left(21 b^2 c^2-24 a b c d+8 a^2 d^2\right) \operatorname{ArcTan}\left[\frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{4 c^4 (b c-a d)^{7/2}}-\frac{3 (b c+2 a d) \operatorname{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{5/2} c^4}$$

Result (type 3, 385 leaves):

$$\frac{1}{8 c^4}\left(\left(2 c \sqrt{a+\frac{b}{x}}\, x \left(-12 b^4 c^3 (d+c x)^2-4 a b^3 c^2 (-3 d+c x) (d+c x)^2+2 a^4 d^3 x \left(6 d^2+9 c d x+2 c^2 x^2\right)+a^3 b d^2 \left(12 d^3-9 c d^2 x-37 c^2 d x^2-12 c^3 x^3\right)+a^2 b^2 c d \left(-27 d^3-29 c d^2 x+12 c^2 d x^2+12 c^3 x^3\right)\right)\right)\Big/\left(a^2 (-b c+a d)^3 (b+a x) (d+c x)^2\right)-\frac{12 (b c+2 a d) \operatorname{Log}\left[b+2 a x+2 \sqrt{a} \sqrt{a+\frac{b}{x}}\, x\right]}{a^{5/2}}+\frac{1}{(b c-a d)^{7/2}} 3\, i\, d^{5/2} \left(21 b^2 c^2-24 a b c d+8 a^2 d^2\right) \operatorname{Log}\left[-\left(\left(8\, i\, c^5 (b c-a d)^{5/2} \left(-b d+b c x-2 a d x-2\, i \sqrt{d} \sqrt{b c-a d} \sqrt{a+\frac{b}{x}}\, x\right)\right)\Big/\left(3 d^{7/2} \left(21 b^2 c^2-24 a b c d+8 a^2 d^2\right) (d+c x)\right)\right)\right]\right)$$

Problem 165: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2}\, dx$$

Optimal (type 3, 287 leaves, 10 steps):

$$\frac{b \left(5 b^2 c^2-6 a b c d+6 a^2 d^2\right)}{3 a^2 c^2 (b c-a d)^2 \left(a+\frac{b}{x}\right)^{3/2}}+\frac{b (b c-2 a d) \left(5 b^2 c^2-a b c d+a^2 d^2\right)}{a^3 c^2 (b c-a d)^3 \sqrt{a+\frac{b}{x}}}+\frac{d (b c-2 a d)}{a c^2 (b c-a d) \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)}+\frac{x}{a c \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)}-\frac{d^{7/2} (9 b c-4 a d) \operatorname{ArcTan}\left[\frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{c^3 (b c-a d)^{7/2}}-\frac{(5 b c+4 a d) \operatorname{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{7/2} c^3}$$

Result (type 3, 364 leaves):

$$\frac{1}{6 c^3}\left(\left(2 \sqrt{a+\frac{b}{x}} \left(3 a^4 d^5 (b+a x)^2+2 b^5 c^3 (b c-a d) (d+c x)-4 b^4 c^3 (4 b c-7 a d) (b+a x) (d+c x)+14 b^4 c^4 (b+a x)^2 (d+c x)-26 a b^3 c^3 d (b+a x)^2 (d+c x)-3 a^4 d^4 (b+a x)^2 (d+c x)+3 a c (b c-a d)^3 x (b+a x)^2 (d+c x)\right)\right) \Big/ \left(a^4 (b c-a d)^3 (b+a x)^2 (d+c x)\right)-\frac{3 (5 b c+4 a d) \operatorname{Log}\left[b+2 a x+2 \sqrt{a} \sqrt{a+\frac{b}{x}} x\right]}{a^{7/2}}+\frac{1}{(b c-a d)^{7/2}} 3 i d^{7/2} (-9 b c+4 a d) \operatorname{Log}\left[\left(2 c^4 (b c-a d)^{5/2} \left(-2 i a d x+2 \sqrt{d} \sqrt{b c-a d} \sqrt{a+\frac{b}{x}} x-i b (d-c x)\right)\right) \Big/ \left(d^{9/2} (9 b c-4 a d) (d+c x)\right)\right]\right)$$

Problem 166: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3} \, dx$$

Optimal (type 3, 409 leaves, 11 steps):

$$\frac{b\left(20 b^3 c^3-36 a b^2 c^2 d+87 a^2 b c d^2-36 a^3 d^3\right)}{12 a^2 c^3 (b c-a d)^3\left(a+\frac{b}{x}\right)^{3/2}}+\frac{b\left(20 b^4 c^4-56 a b^3 c^3 d+24 a^2 b^2 c^2 d^2-35 a^3 b c d^3+12 a^4 d^4\right)}{4 a^3 c^3 (b c-a d)^4 \sqrt{a+\frac{b}{x}}}+$$

$$\frac{d (2 b c-3 a d)}{2 a c^2 (b c-a d)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}+\frac{d\left(4 b^2 c^2-23 a b c d+12 a^2 d^2\right)}{4 a c^3 (b c-a d)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}+\frac{x}{a c\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}-$$

$$\frac{d^{7/2}\left(99 b^2 c^2-88 a b c d+24 a^2 d^2\right) \text{ArcTan}\left[\frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{b c-a d}}\right]}{4 c^4 (b c-a d)^{9/2}}-\frac{(5 b c+6 a d) \text{ArcTanh}\left[\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right]}{a^{7/2} c^4}$$

Result (type 3, 465 leaves):

$$-\frac{1}{24 c^4}\left(\frac{1}{a^4 (b c-a d)^4 (b+a x)^2 (d+c x)^2}\right.$$

$$2 \sqrt{a+\frac{b}{x}}\left(6 a^4 d^6 (b c-a d)(b+a x)^2+3 a^4 d^5(-23 b c+12 a d)(b+a x)^2(d+c x)-\right.$$
$$8 b^6 c^4 (b c-a d)(d+c x)^2+8 b^5 c^4(8 b c-17 a d)(b+a x)(d+c x)^2-$$
$$56 b^5 c^5 (b+a x)^2 (d+c x)^2+128 a b^4 c^4 d (b+a x)^2 (d+c x)^2+63 a^4 b c d^4 (b+a x)^2$$
$$\left.(d+c x)^2-30 a^5 d^5 (b+a x)^2 (d+c x)^2-12 a c (b c-a d)^4 x (b+a x)^2 (d+c x)^2\right)+$$

$$\frac{12 (5 b c+6 a d) \text{Log}\left[b+2 a x+2 \sqrt{a} \sqrt{a+\frac{b}{x}}\, x\right]}{a^{7/2}}+\frac{1}{(b c-a d)^{9/2}}$$
$$3 i d^{7/2}\left(99 b^2 c^2-88 a b c d+24 a^2 d^2\right)$$
$$\text{Log}\left[\left(8 c^5 (b c-a d)^{7/2}\left(-2 i a d x+2 \sqrt{d} \sqrt{b c-a d} \sqrt{a+\frac{b}{x}}\, x-i b (d-c x)\right)\right)\Big/\right.$$
$$\left.\left.\left(d^{9/2}\left(99 b^2 c^2-88 a b c d+24 a^2 d^2\right)(d+c x)\right)\right]\right)$$

Problem 170: Result more than twice size of optimal antiderivative.

$$\int\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q dx$$

Optimal (type 6, 96 leaves, 3 steps):

$$-\frac{1}{a^2\,(1+p)}b\left(a+\frac{b}{x}\right)^{1+p}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{b\,c-a\,d}\right)^{-q}\text{AppellF1}\left[1+p,\,-q,\,2,\,2+p,\,-\frac{d\left(a+\frac{b}{x}\right)}{b\,c-a\,d},\,\frac{a+\frac{b}{x}}{a}\right]$$

Result (type 6, 206 leaves):

$$\left(b\,d\,(-2+p+q)\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q x\,\text{AppellF1}\left[1-p-q,\,-p,\,-q,\,2-p-q,\,-\frac{a\,x}{b},\,-\frac{c\,x}{d}\right]\right)\Big/ \left((-1+p+q)\left(-b\,d\,(-2+p+q)\,\text{AppellF1}\left[1-p-q,\,-p,\,-q,\,2-p-q,\,-\frac{a\,x}{b},\,-\frac{c\,x}{d}\right]+x\left(a\,d\,p\,\text{AppellF1}\left[2-p-q,\,1-p,\,-q,\,3-p-q,\,-\frac{a\,x}{b},\,-\frac{c\,x}{d}\right]+b\,c\,q\,\text{AppellF1}\left[2-p-q,\,-p,\,1-q,\,3-p-q,\,-\frac{a\,x}{b},\,-\frac{c\,x}{d}\right]\right)\right)\right)$$

Problem 172: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}\,dx$$

Optimal (type 4, 233 leaves, 6 steps):

$$-\frac{2\,d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}\,x}+\sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}\,x+\frac{2\sqrt{c}\,\sqrt{d}\,\sqrt{a+\frac{b}{x^2}}\,\text{EllipticE}\left[\text{ArcCot}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right],\,1-\frac{b\,c}{a\,d}\right]}{\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\,\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c}\,(b\,c+a\,d)\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left[\text{ArcCot}\left[\frac{\sqrt{c}\,x}{\sqrt{d}}\right],\,1-\frac{b\,c}{a\,d}\right]}{a\sqrt{d}\,\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\,\sqrt{c+\frac{d}{x^2}}}$$

Result (type 4, 205 leaves):

$$-\left(\left(\sqrt{a+\frac{b}{x^2}}\,\sqrt{c+\frac{d}{x^2}}\,x\left(\sqrt{\frac{a}{b}}\,(b+a\,x^2)\,(d+c\,x^2)+2\,i\,a\,d\,x\sqrt{1+\frac{a\,x^2}{b}}\,\sqrt{1+\frac{c\,x^2}{d}}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{a}{b}}\,x\right],\,\frac{b\,c}{a\,d}\right]+i\,(b\,c-a\,d)\,x\sqrt{1+\frac{a\,x^2}{b}}\,\sqrt{1+\frac{c\,x^2}{d}}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{\frac{a}{b}}\,x\right],\,\frac{b\,c}{a\,d}\right]\right)\right)\Big/\left(\sqrt{\frac{a}{b}}\,(b+a\,x^2)\,(d+c\,x^2)\right)\right)$$

Problem 174: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

Optimal (type 4, 262 leaves, 7 steps):

$$-\frac{2 d \sqrt{a+\frac{b}{x^2}}}{c^2 \sqrt{c+\frac{d}{x^2}}\, x}-\frac{\sqrt{a+\frac{b}{x^2}}\, x}{c \sqrt{c+\frac{d}{x^2}}}+\frac{2 \sqrt{a+\frac{b}{x^2}} \sqrt{c+\frac{d}{x^2}}\, x}{c^2}+\frac{2 \sqrt{d} \sqrt{a+\frac{b}{x^2}}\, \text{EllipticE}\left[\text{ArcCot}\left[\frac{\sqrt{c}\, x}{\sqrt{d}}\right], 1-\frac{b c}{a d}\right]}{c^{3/2} \sqrt{\frac{c \left(a+\frac{b}{x^2}\right)}{a \left(c+\frac{d}{x^2}\right)}} \sqrt{c+\frac{d}{x^2}}}-\frac{b \sqrt{a+\frac{b}{x^2}}\, \text{EllipticF}\left[\text{ArcCot}\left[\frac{\sqrt{c}\, x}{\sqrt{d}}\right], 1-\frac{b c}{a d}\right]}{a \sqrt{c} \sqrt{d} \sqrt{\frac{c \left(a+\frac{b}{x^2}\right)}{a \left(c+\frac{d}{x^2}\right)}} \sqrt{c+\frac{d}{x^2}}}$$

Result (type 4, 191 leaves):

$$-\left(\left(\sqrt{a+\frac{b}{x^2}} \left(\sqrt{\frac{a}{b}}\, c\, x \left(b+a x^2\right)+2 i a d \sqrt{1+\frac{a x^2}{b}} \sqrt{1+\frac{c x^2}{d}}\, \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{a}{b}}\, x\right], \frac{b c}{a d}\right]+i (b c-2 a d) \sqrt{1+\frac{a x^2}{b}} \sqrt{1+\frac{c x^2}{d}}\, \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{a}{b}}\, x\right], \frac{b c}{a d}\right]\right)\right) \Big/ \left(\sqrt{\frac{a}{b}}\, c^2 \sqrt{c+\frac{d}{x^2}} \left(b+a x^2\right)\right)\right)$$

Problem 175: Result more than twice size of optimal antiderivative.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Optimal (type 6, 79 leaves, 4 steps):

$$\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{a x^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{c x^2}\right)^{-q} x\, \text{AppellF1}\left[-\frac{1}{2}, -p, -q, \frac{1}{2}, -\frac{b}{a x^2}, -\frac{d}{c x^2}\right]$$

Result (type 6, 252 leaves):

$$\left(b\, d\, (-3+2 p+2 q) \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\, \text{AppellF1}\left[\frac{1}{2}-p-q,-p,-q,\frac{3}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right) \Big/ \left((-1+2 p+2 q) \left(b\, d\, (3-2 p-2 q)\, \text{AppellF1}\left[\frac{1}{2}-p-q,-p,-q,\frac{3}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+2 x^2 \left(a\, d\, p\, \text{AppellF1}\left[\frac{3}{2}-p-q,1-p,-q,\frac{5}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]+b\, c\, q\, \text{AppellF1}\left[\frac{3}{2}-p-q,-p,1-q,\frac{5}{2}-p-q,-\frac{a x^2}{b},-\frac{c x^2}{d}\right]\right)\right)\right)$$

Problem 213: Result more than twice size of optimal antiderivative.

$$\int (a+b x^n)^p (c+d x^n)^q \, dx$$

Optimal (type 6, 81 leaves, 3 steps):

$$x (a+b x^n)^p \left(1+\frac{b x^n}{a}\right)^{-p} (c+d x^n)^q \left(1+\frac{d x^n}{c}\right)^{-q} \text{AppellF1}\left[\frac{1}{n},-p,-q,1+\frac{1}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]$$

Result (type 6, 190 leaves):

$$\left(a\, c\, (1+n)\, x\, (a+b x^n)^p (c+d x^n)^q\, \text{AppellF1}\left[\frac{1}{n},-p,-q,1+\frac{1}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]\right) \Big/ \left(b\, c\, n\, p\, x^n\, \text{AppellF1}\left[1+\frac{1}{n},1-p,-q,2+\frac{1}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]+a\, d\, n\, q\, x^n\, \text{AppellF1}\left[1+\frac{1}{n},-p,1-q,2+\frac{1}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]+a\, c\, (1+n)\, \text{AppellF1}\left[\frac{1}{n},-p,-q,1+\frac{1}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]\right)$$

Problem 218: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x^n)^p}{c+d x^n} \, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x (a+b x^n)^p \left(1+\frac{b x^n}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{n},-p,1,1+\frac{1}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]}{c}$$

Result (type 6, 180 leaves):

$$\left(a\, c\, (1+n)\, x\, (a+b x^n)^p\, \text{AppellF1}\left[\frac{1}{n},-p,1,1+\frac{1}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]\right) \Big/ \left((c+d x^n) \left(b\, c\, n\, p\, x^n\, \text{AppellF1}\left[1+\frac{1}{n},1-p,1,2+\frac{1}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]-a\, d\, n\, x^n\, \text{AppellF1}\left[1+\frac{1}{n},-p,2,2+\frac{1}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]+a\, c\, (1+n)\, \text{AppellF1}\left[\frac{1}{n},-p,1,1+\frac{1}{n},-\frac{b x^n}{a},-\frac{d x^n}{c}\right]\right)\right)$$

Problem 219: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x^n)^p}{(c+d x^n)^2} \, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x (a+b x^n)^p \left(1+\frac{b x^n}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{n}, -p, 2, 1+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]}{c^2}$$

Result (type 6, 180 leaves):

$$\left(a c (1+n) x (a+b x^n)^p \text{AppellF1}\left[\frac{1}{n}, -p, 2, 1+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]\right) \Big/ \left((c+d x^n)^2 \left(b c n p x^n \text{AppellF1}\left[1+\frac{1}{n}, 1-p, 2, 2+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right] - 2 a d n x^n \text{AppellF1}\left[1+\frac{1}{n}, -p, 3, 2+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right] + a c (1+n) \text{AppellF1}\left[\frac{1}{n}, -p, 2, 1+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]\right)\right)$$

Problem 220: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b x^n)^p}{(c+d x^n)^3} \, dx$$

Optimal (type 6, 59 leaves, 2 steps):

$$\frac{x (a+b x^n)^p \left(1+\frac{b x^n}{a}\right)^{-p} \text{AppellF1}\left[\frac{1}{n}, -p, 3, 1+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]}{c^3}$$

Result (type 6, 180 leaves):

$$\left(a c (1+n) x (a+b x^n)^p \text{AppellF1}\left[\frac{1}{n}, -p, 3, 1+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]\right) \Big/ \left((c+d x^n)^3 \left(b c n p x^n \text{AppellF1}\left[1+\frac{1}{n}, 1-p, 3, 2+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right] - 3 a d n x^n \text{AppellF1}\left[1+\frac{1}{n}, -p, 4, 2+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right] + a c (1+n) \text{AppellF1}\left[\frac{1}{n}, -p, 3, 1+\frac{1}{n}, -\frac{b x^n}{a}, -\frac{d x^n}{c}\right]\right)\right)$$

Problem 222: Result unnecessarily involves higher level functions.

$$\int (a+b x^n)^3 (c+d x^n)^{-4-\frac{1}{n}} \, dx$$

Optimal (type 3, 178 leaves, 4 steps):

$$\frac{x\left(a+b x^n\right)^3\left(c+d x^n\right)^{-3-\frac{1}{n}}}{c\left(1+3 n\right)}+\frac{3 a n x\left(a+b x^n\right)^2\left(c+d x^n\right)^{-2-\frac{1}{n}}}{c^2\left(1+5 n+6 n^2\right)}+$$
$$\frac{6 a^2 n^2 x\left(a+b x^n\right)\left(c+d x^n\right)^{-1-\frac{1}{n}}}{c^3\left(1+n\right)\left(1+2 n\right)\left(1+3 n\right)}+\frac{6 a^3 n^3 x\left(c+d x^n\right)^{-1/n}}{c^4\left(1+n\right)\left(1+2 n\right)\left(1+3 n\right)}$$

Result (type 5, 198 leaves):

$$\frac{1}{c^4}x\left(c+d x^n\right)^{-1/n}$$
$$\left(\frac{b^3 c^3 x^{3 n}}{\left(1+3 n\right)\left(c+d x^n\right)^3}+\frac{3 a^2 b x^n\left(1+\frac{d x^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left[1+\frac{1}{n},4+\frac{1}{n},2+\frac{1}{n},-\frac{d x^n}{c}\right]}{1+n}+\right.$$
$$\frac{3 a b^2 x^{2 n}\left(1+\frac{d x^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left[2+\frac{1}{n},4+\frac{1}{n},3+\frac{1}{n},-\frac{d x^n}{c}\right]}{1+2 n}+$$
$$\left.a^3\left(1+\frac{d x^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left[4+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{d x^n}{c}\right]\right)$$

Problem 223: Result unnecessarily involves higher level functions.

$$\int\left(a+b x^n\right)^2\left(c+d x^n\right)^{-3-\frac{1}{n}}\,dx$$

Optimal (type 3, 116 leaves, 3 steps):

$$\frac{x\left(a+b x^n\right)^2\left(c+d x^n\right)^{-2-\frac{1}{n}}}{c\left(1+2 n\right)}+\frac{2 a n x\left(a+b x^n\right)\left(c+d x^n\right)^{-1-\frac{1}{n}}}{c^2\left(1+n\right)\left(1+2 n\right)}+\frac{2 a^2 n^2 x\left(c+d x^n\right)^{-1/n}}{c^3\left(1+n\right)\left(1+2 n\right)}$$

Result (type 5, 139 leaves):

$$\frac{1}{c^3}x\left(c+d x^n\right)^{-1/n}$$
$$\left(\frac{b^2 c^2 x^{2 n}}{\left(1+2 n\right)\left(c+d x^n\right)^2}+\frac{2 a b x^n\left(1+\frac{d x^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left[1+\frac{1}{n},3+\frac{1}{n},2+\frac{1}{n},-\frac{d x^n}{c}\right]}{1+n}+\right.$$
$$\left.a^2\left(1+\frac{d x^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left[3+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{d x^n}{c}\right]\right)$$

Problem 224: Result unnecessarily involves higher level functions.

$$\int\left(a+b x^n\right)\left(c+d x^n\right)^{-2-\frac{1}{n}}\,dx$$

Optimal (type 3, 58 leaves, 2 steps):

$$\frac{x\left(a+b x^n\right)\left(c+d x^n\right)^{-1-\frac{1}{n}}}{c\,(1+n)}+\frac{a\,n\,x\left(c+d x^n\right)^{-1/n}}{c^2\,(1+n)}$$

Result (type 5, 82 leaves):

$$\frac{1}{c^2\,(1+n)}x\left(c+d x^n\right)^{-\frac{1+n}{n}}\left(b\,c\,x^n+a\,(1+n)\left(c+d x^n\right)\left(1+\frac{d x^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left[2+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{d x^n}{c}\right]\right)$$

Problem 228: Attempted integration timed out after 120 seconds.

$$\int\frac{\left(c+d x^n\right)^{2-\frac{1}{n}}}{\left(a+b x^n\right)^3}\,dx$$

Optimal (type 5, 56 leaves, 1 step):

$$\frac{c^2\,x\left(c+d x^n\right)^{-1/n}\text{Hypergeometric2F1}\left[3,\frac{1}{n},1+\frac{1}{n},-\frac{(b c-a d)\,x^n}{a\left(c+d x^n\right)}\right]}{a^3}$$

Result (type 1, 1 leaves):

???

Problem 229: Result more than twice size of optimal antiderivative.

$$\int\left(a+b x^n\right)^p\left(c+d x^n\right)^{-2-\frac{1}{n}-p}\,dx$$

Optimal (type 5, 193 leaves, 2 steps):

$$-\frac{b\,x\left(a+b x^n\right)^{1+p}\left(c+d x^n\right)^{-1-\frac{1}{n}-p}}{a\,(b c-a d)\,n\,(1+p)}+$$
$$\left(\left(b\,c+(b c-a d)\,n\,(1+p)\right)x\left(a+b x^n\right)^{1+p}\left(\frac{c\left(a+b x^n\right)}{a\left(c+d x^n\right)}\right)^{-1-p}\left(c+d x^n\right)^{-1-\frac{1}{n}-p}\right.$$
$$\left.\text{Hypergeometric2F1}\left[\frac{1}{n},-1-p,1+\frac{1}{n},-\frac{(b c-a d)\,x^n}{a\left(c+d x^n\right)}\right]\right)\Big/\left(a\,c\,(b c-a d)\,n\,(1+p)\right)$$

Result (type 5, 1414 leaves):

$$\left(c^4\,(1+n)\,(1+2 n)\,(1+3 n)\,x\left(a+b x^n\right)^{3+p}\left(c+d x^n\right)^{-2-\frac{1}{n}-p}\left(1+\frac{d x^n}{c}\right)\right.$$
$$\text{Gamma}\left[2+\frac{1}{n}\right]\text{Gamma}[-p]\left(\text{Hypergeometric2F1}\left[1,-p,1+\frac{1}{n},\frac{(b c-a d)\,x^n}{c\left(a+b x^n\right)}\right]+\right.$$

$$\frac{1}{c^2} d\, n\, x^n \left(\frac{c\, \text{Hypergeometric2F1}\left[1, -p, 2+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]}{1+n} + \right.$$

$$\left(\left(b\, c-a\, d\right) x^n\, \text{Gamma}\left[1+\frac{1}{n}\right] \text{Gamma}[1-p]\, \text{Hypergeometric2F1}\left[2, 1-p, 3+\frac{1}{n},\right.\right.$$

$$\left.\left.\left.\left.\frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]\right) \Big/ \left(\left(1+2\, n\right) \left(a+b\, x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Gamma}[-p]\right)\right)\right)\right) \Big/$$

$$\left(-c\, d\, (1+3\, n)\, (1+n+n\, p)\, x^n\, (a+b\, x^n)^2 \left(c^2\, (1+n)\, (1+2\, n)\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right] \text{Gamma}[-p]\right.\right.$$

$$\text{Hypergeometric2F1}\left[1, -p, 1+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right] + d\, n\, x^n \left(c\, (1+2\, n)\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right]\right.$$

$$\text{Gamma}[-p]\, \text{Hypergeometric2F1}\left[1, -p, 2+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right] + (b\, c-a\, d)\, (1+n)\, x^n$$

$$\left.\left.\text{Gamma}\left[1+\frac{1}{n}\right] \text{Gamma}[1-p]\, \text{Hypergeometric2F1}\left[2, 1-p, 3+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]\right)\right) +$$

$$b\, c\, n\, (1+3\, n)\, p\, x^n\, (a+b\, x^n)\, (c+d\, x^n) \left(c^2\, (1+n)\, (1+2\, n)\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right]\right.$$

$$\text{Gamma}[-p]\, \text{Hypergeometric2F1}\left[1, -p, 1+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right] +$$

$$d\, n\, x^n \left(c\, (1+2\, n)\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right] \text{Gamma}[-p]\right.$$

$$\text{Hypergeometric2F1}\left[1, -p, 2+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right] + (b\, c-a\, d)\, (1+n)\, x^n\, \text{Gamma}\left[1+\frac{1}{n}\right]$$

$$\left.\left.\text{Gamma}[1-p]\, \text{Hypergeometric2F1}\left[2, 1-p, 3+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]\right)\right) +$$

$$c\, (1+3\, n)\, (a+b\, x^n)^2\, (c+d\, x^n) \left(c^2\, (1+n)\, (1+2\, n)\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right] \text{Gamma}[-p]\right.$$

$$\text{Hypergeometric2F1}\left[1, -p, 1+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right] + d\, n\, x^n \left(c\, (1+2\, n)\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right]\right.$$

$$\text{Gamma}[-p]\, \text{Hypergeometric2F1}\left[1, -p, 2+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right] + (b\, c-a\, d)\, (1+n)\, x^n$$

$$\left.\left.\text{Gamma}\left[1+\frac{1}{n}\right] \text{Gamma}[1-p]\, \text{Hypergeometric2F1}\left[2, 1-p, 3+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right]\right)\right) +$$

$$n^2\, x^n\, (c+d\, x^n) \left(a\, c^2\, (-b\, c+a\, d)\, (1+2\, n)\, (1+3\, n)\, p\, (a+b\, x^n)\, \text{Gamma}\left[2+\frac{1}{n}\right]\right.$$

$$\text{Gamma}[-p]\, \text{Hypergeometric2F1}\left[2, 1-p, 2+\frac{1}{n}, \frac{(b\, c-a\, d)\, x^n}{c\, (a+b\, x^n)}\right] +$$

$$c\,d\,(1+3n)\,(a+b\,x^n)^2\Bigg(c\,(1+2n)\,(a+b\,x^n)\,\text{Gamma}\Big[2+\frac{1}{n}\Big]\,\text{Gamma}[-p]$$
$$\text{Hypergeometric2F1}\Big[1,\,-p,\,2+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\Big]+(b\,c-a\,d)\,(1+n)\,x^n$$
$$\text{Gamma}\Big[1+\frac{1}{n}\Big]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\Big[2,\,1-p,\,3+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\Big]\Bigg)-$$
$$d\,(b\,c-a\,d)\,x^n\Bigg(b\,c\,(1+n)\,(1+3n)\,x^n\,(a+b\,x^n)\,\text{Gamma}\Big[1+\frac{1}{n}\Big]\,\text{Gamma}[1-p]$$
$$\text{Hypergeometric2F1}\Big[2,\,1-p,\,3+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\Big]-c\,(1+n)\,(1+3n)\,(a+b\,x^n)^2$$
$$\text{Gamma}\Big[1+\frac{1}{n}\Big]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\Big[2,\,1-p,\,3+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\Big]+$$
$$a\,c\,n\,(1+3n)\,p\,(a+b\,x^n)\,\text{Gamma}\Big[2+\frac{1}{n}\Big]\,\text{Gamma}[-p]$$
$$\text{Hypergeometric2F1}\Big[2,\,1-p,\,3+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\Big]-2\,a\,(-b\,c+a\,d)\,n\,(1+n)\,(-1+p)$$
$$x^n\,\text{Gamma}\Big[1+\frac{1}{n}\Big]\,\text{Gamma}[1-p]\,\text{Hypergeometric2F1}\Big[3,\,2-p,\,4+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\Big]\Bigg)\Bigg)\Bigg)$$

Problem 230: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a+b\,x^n)^{\frac{a\,d\,n-b\,c\,(1+n)}{(b\,c-a\,d)\,n}}\,(c+d\,x^n)^{\frac{a\,d-b\,c\,n+a\,d\,n}{b\,c\,n-a\,d\,n}}\,dx$$

Optimal (type 3, 57 leaves, 1 step):

$$\frac{x\,(a+b\,x^n)^{-\frac{b\,c}{(b\,c-a\,d)\,n}}\,(c+d\,x^n)^{\frac{a\,d}{(b\,c-a\,d)\,n}}}{a\,c}$$

Result (type 6, 461 leaves):

$$\left(a\,c\,(-b\,c+a\,d)\,(1+n)\,x\,(a+b\,x^n)^{\frac{a\,d\,n-b\,c\,(1+n)}{(b\,c-a\,d)\,n}}\,(c+d\,x^n)^{\frac{a\,d-b\,c\,n+a\,d\,n}{b\,c\,n-a\,d\,n}}\right.$$
$$\left.\text{AppellF1}\left[\frac{1}{n},\frac{b\,c+b\,c\,n-a\,d\,n}{b\,c\,n-a\,d\,n},\frac{b\,c\,n-a\,d\,(1+n)}{(b\,c-a\,d)\,n},1+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\Big/$$
$$\left(b\,c\,(-a\,d\,n+b\,c\,(1+n))\,x^n\,\text{AppellF1}\left[1+\frac{1}{n},\frac{b\,c+2\,b\,c\,n-2\,a\,d\,n}{b\,c\,n-a\,d\,n},\frac{b\,c\,n-a\,d\,(1+n)}{(b\,c-a\,d)\,n},\right.\right.$$
$$\left.2+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]-a\left(d\,(-b\,c\,n+a\,d\,(1+n))\,x^n\,\text{AppellF1}\left[1+\frac{1}{n},\frac{b\,c+b\,c\,n-a\,d\,n}{b\,c\,n-a\,d\,n},\right.\right.$$
$$\left.-\frac{a\,d-2\,b\,c\,n+2\,a\,d\,n}{b\,c\,n-a\,d\,n},2+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]+c\,(b\,c-a\,d)\,(1+n)$$
$$\left.\left.\text{AppellF1}\left[\frac{1}{n},\frac{b\,c+b\,c\,n-a\,d\,n}{b\,c\,n-a\,d\,n},\frac{b\,c\,n-a\,d\,(1+n)}{(b\,c-a\,d)\,n},1+\frac{1}{n},-\frac{b\,x^n}{a},-\frac{d\,x^n}{c}\right]\right)\right)$$

Problem 231: Result unnecessarily involves higher level functions.

$$\int (a+b\,x^n)^2\,(c+d\,x^n)^{-4-\frac{1}{n}}\,dx$$

Optimal (type 3, 327 leaves, 5 steps):

$$-\frac{b\,x\,(a+b\,x^n)^3\,(c+d\,x^n)^{-3-\frac{1}{n}}}{3\,a\,(b\,c-a\,d)\,n}-\frac{(3\,a\,d\,n-b\,(c+3\,c\,n))\,x\,(a+b\,x^n)^3\,(c+d\,x^n)^{-3-\frac{1}{n}}}{3\,a\,c\,(b\,c-a\,d)\,n\,(1+3\,n)}-$$
$$\frac{(3\,a\,d\,n-b\,(c+3\,c\,n))\,x\,(a+b\,x^n)^2\,(c+d\,x^n)^{-2-\frac{1}{n}}}{c^2\,(b\,c-a\,d)\,(1+5\,n+6\,n^2)}-$$
$$\frac{2\,a\,n\,(3\,a\,d\,n-b\,(c+3\,c\,n))\,x\,(a+b\,x^n)\,(c+d\,x^n)^{-1-\frac{1}{n}}}{c^3\,(b\,c-a\,d)\,(1+n)\,(1+2\,n)\,(1+3\,n)}-\frac{2\,a^2\,n^2\,(3\,a\,d\,n-b\,(c+3\,c\,n))\,x\,(c+d\,x^n)^{-1/n}}{c^4\,(b\,c-a\,d)\,(1+n)\,(1+2\,n)\,(1+3\,n)}$$

Result (type 5, 153 leaves):

$$\left(x\,(c+d\,x^n)^{-1/n}\left(1+\frac{d\,x^n}{c}\right)^{\frac{1}{n}}\left(2\,a\,b\,(1+2\,n)\,x^n\,\text{Hypergeometric2F1}\left[1+\frac{1}{n},4+\frac{1}{n},2+\frac{1}{n},-\frac{d\,x^n}{c}\right]+\right.\right.$$
$$(1+n)\left(b^2\,x^{2\,n}\,\text{Hypergeometric2F1}\left[2+\frac{1}{n},4+\frac{1}{n},3+\frac{1}{n},-\frac{d\,x^n}{c}\right]+\right.$$
$$\left.\left.\left.a^2\,(1+2\,n)\,\text{Hypergeometric2F1}\left[4+\frac{1}{n},\frac{1}{n},1+\frac{1}{n},-\frac{d\,x^n}{c}\right]\right)\right)\right)\Big/\left(c^4\,(1+n)\,(1+2\,n)\right)$$

Problem 232: Result unnecessarily involves higher level functions.

$$\int (a+b\,x^n)\,(c+d\,x^n)^{-3-\frac{1}{n}}\,dx$$

Optimal (type 3, 127 leaves, 3 steps):

$$-\frac{(b\,c-a\,d)\,x\,(c+d\,x^n)^{-2-\frac{1}{n}}}{c\,d\,(1+2\,n)}+\frac{(b\,c+2\,a\,d\,n)\,x\,(c+d\,x^n)^{-1-\frac{1}{n}}}{c^2\,d\,(1+n)\,(1+2\,n)}+\frac{n\,(b\,c+2\,a\,d\,n)\,x\,(c+d\,x^n)^{-1/n}}{c^3\,d\,(1+n)\,(1+2\,n)}$$

Result (type 5, 96 leaves):

$$\frac{1}{c^3\,(1+n)}x\,(c+d\,x^n)^{-1/n}\left(1+\frac{d\,x^n}{c}\right)^{\frac{1}{n}}\left(b\,x^n\,\text{Hypergeometric2F1}\left[1+\frac{1}{n},\,3+\frac{1}{n},\,2+\frac{1}{n},\,-\frac{d\,x^n}{c}\right]+a\,(1+n)\,\text{Hypergeometric2F1}\left[3+\frac{1}{n},\,\frac{1}{n},\,1+\frac{1}{n},\,-\frac{d\,x^n}{c}\right]\right)$$

Problem 233: Result unnecessarily involves higher level functions.

$$\int (c+d\,x^n)^{-2-\frac{1}{n}}\,dx$$

Optimal (type 3, 50 leaves, 2 steps):

$$\frac{x\,(c+d\,x^n)^{-1-\frac{1}{n}}}{c\,(1+n)}+\frac{n\,x\,(c+d\,x^n)^{-1/n}}{c^2\,(1+n)}$$

Result (type 5, 55 leaves):

$$\frac{x\,(c+d\,x^n)^{-1/n}\left(1+\frac{d\,x^n}{c}\right)^{\frac{1}{n}}\text{Hypergeometric2F1}\left[2+\frac{1}{n},\,\frac{1}{n},\,1+\frac{1}{n},\,-\frac{d\,x^n}{c}\right]}{c^2}$$

Problem 235: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+d\,x^n)^{-1/n}}{(a+b\,x^n)^2}\,dx$$

Optimal (type 5, 127 leaves, 2 steps):

$$\frac{b\,x\,(c+d\,x^n)^{-\frac{1-n}{n}}}{a\,(b\,c-a\,d)\,n\,(a+b\,x^n)}-\frac{1}{a^2\,(b\,c-a\,d)\,n}(b\,c\,(1-n)+a\,d\,n)\,x\,(c+d\,x^n)^{-1/n}\,\text{Hypergeometric2F1}\left[1,\,\frac{1}{n},\,1+\frac{1}{n},\,-\frac{(b\,c-a\,d)\,x^n}{a\,(c+d\,x^n)}\right]$$

Result (type 5, 1070 leaves):

$$\left(c^2\,(1+2\,n)\,(1+3\,n)\,x\,(a+b\,x^n)\,(c+d\,x^n)^{-1/n}\left(1+\frac{d\,x^n}{c}\right)\text{Gamma}\left[2+\frac{1}{n}\right]\right.$$

$$\text{Gamma}\left[3+\frac{1}{n}\right]\left(\frac{c\,(c+c\,n+d\,n\,x^n)\,\text{Hypergeometric2F1}\left[1,\,2,\,2+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]}{\text{Gamma}\left[2+\frac{1}{n}\right]}+\right.$$

$$\left.\left(2\,(b\,c-a\,d)\,n\,x^n\,(c+d\,x^n)\,\text{Hypergeometric2F1}\left[2,\,3,\,3+\frac{1}{n},\,\frac{(b\,c-a\,d)\,x^n}{c\,(a+b\,x^n)}\right]\right)\right/$$

$$\left(\left(a+b x^n\right) \text{Gamma}\left[3+\frac{1}{n}\right]\right)\Bigg)\Bigg) \Big/ \Bigg(-c\, d\, (1-n)\, (1+2 n)\, (1+3 n)\, x^n \left(a+b x^n\right)^2$$

$$\left(c \left(a+b x^n\right) \left(c+c n+d n x^n\right) \text{Gamma}\left[3+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[1,2,2+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+\right.$$

$$\left.2 (b c-a d)\, n\, x^n \left(c+d x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]\right)-$$

$$2 b c n (1+2 n) (1+3 n) x^n \left(a+b x^n\right) \left(c+d x^n\right)$$

$$\left(c \left(a+b x^n\right) \left(c+c n+d n x^n\right) \text{Gamma}\left[3+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[1,2,2+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+\right.$$

$$\left.2 (b c-a d)\, n\, x^n \left(c+d x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]\right)+$$

$$c (1+2 n) (1+3 n) \left(a+b x^n\right)^2 \left(c+d x^n\right)$$

$$\left(c \left(a+b x^n\right) \left(c+c n+d n x^n\right) \text{Gamma}\left[3+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[1,2,2+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+\right.$$

$$\left.2 (b c-a d)\, n\, x^n \left(c+d x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]\right)+$$

$$n^2 x^n \left(c+d x^n\right) \left(c^2 d (1+2 n) (1+3 n) \left(a+b x^n\right)^3 \text{Gamma}\left[3+\frac{1}{n}\right]\right.$$

$$\text{Hypergeometric2F1}\left[1,2,2+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+2 c d (b c-a d) (1+2 n) (1+3 n)$$

$$x^n \left(a+b x^n\right)^2 \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]-$$

$$2 b c (b c-a d) (1+2 n) (1+3 n) x^n \left(a+b x^n\right) \left(c+d x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right]$$

$$\text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+2 c (b c-a d) (1+2 n) (1+3 n)$$

$$\left(a+b x^n\right)^2 \left(c+d x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+$$

$$2 a c (b c-a d) (1+3 n) \left(a+b x^n\right) \left(c+c n+d n x^n\right) \text{Gamma}\left[3+\frac{1}{n}\right]$$

$$\text{Hypergeometric2F1}\left[2,3,3+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+12 a (b c-a d)^2 n (1+2 n)$$

$$\left.x^n \left(c+d x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[3,4,4+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]\right)\Bigg)$$

Problem 236: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(c+d x^n\right)^{1-\frac{1}{n}}}{\left(a+b x^n\right)^3} \, dx$$

Optimal (type 5, 131 leaves, 2 steps):

$$\frac{b x \left(c+d x^n\right)^{2-\frac{1}{n}}}{2 a (b c-a d) n \left(a+b x^n\right)^2}-\frac{1}{2 a^3 (b c-a d) n}$$
$$c (b c (1-2 n)+2 a d n) x \left(c+d x^n\right)^{-1/n} \text{Hypergeometric2F1}\left[2,\frac{1}{n},1+\frac{1}{n},-\frac{(b c-a d) x^n}{a \left(c+d x^n\right)}\right]$$

Result (type 5, 1251 leaves):

$$-\Bigg(\Bigg(c^4 (1+n) (1+2 n) (1+3 n) x \left(c+d x^n\right)^{\frac{-1+n}{n}}$$
$$\left(1+\frac{d x^n}{c}\right) \text{Gamma}\left[2+\frac{1}{n}\right] \Bigg(\text{Hypergeometric2F1}\left[1,3,1+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+$$
$$\frac{1}{c^2} d n x^n \left(\frac{c\, \text{Hypergeometric2F1}\left[1,3,2+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]}{1+n}+\right.$$
$$\left(3 (b c-a d) x^n \text{Gamma}\left[1+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,4,3+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]\right)\Big/$$
$$\left.\left((1+2 n) \left(a+b x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right]\right)\right)\Bigg)\Bigg)\Bigg/$$
$$\Bigg(c d (1-2 n) (1+3 n) x^n \left(a+b x^n\right)^2 \Bigg(c^2 (1+n) (1+2 n) \left(a+b x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right]$$
$$\text{Hypergeometric2F1}\left[1,3,1+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+$$
$$d n x^n \Bigg(c (1+2 n) \left(a+b x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[1,3,2+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+$$
$$3 (b c-a d) (1+n) x^n \text{Gamma}\left[1+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,4,3+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]\Bigg)\Bigg)+$$
$$3 b c n (1+3 n) x^n \left(a+b x^n\right) \left(c+d x^n\right) \Bigg(c^2 (1+n) (1+2 n) \left(a+b x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right]$$
$$\text{Hypergeometric2F1}\left[1,3,1+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+$$
$$d n x^n \Bigg(c (1+2 n) \left(a+b x^n\right) \text{Gamma}\left[2+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[1,3,2+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]+$$
$$3 (b c-a d) (1+n) x^n \text{Gamma}\left[1+\frac{1}{n}\right] \text{Hypergeometric2F1}\left[2,4,3+\frac{1}{n},\frac{(b c-a d) x^n}{c \left(a+b x^n\right)}\right]\Bigg)\Bigg)-$$

$$c\,(1+3n)\,(a+b x^n)^2\,(c+d x^n)\,\Big(c^2\,(1+n)\,(1+2n)\,(a+b x^n)\,\text{Gamma}\Big[2+\frac{1}{n}\Big]\,\text{Hypergeometric2F1}\Big[1,3,1+\frac{1}{n},\frac{(b c-a d)\,x^n}{c\,(a+b x^n)}\Big]+d\,n\,x^n\,\Big(c\,(1+2n)\,(a+b x^n)\,\text{Gamma}\Big[2+\frac{1}{n}\Big]\,\text{Hypergeometric2F1}\Big[1,3,2+\frac{1}{n},\frac{(b c-a d)\,x^n}{c\,(a+b x^n)}\Big]+3\,(b c-a d)\,(1+n)\,x^n\,\text{Gamma}\Big[1+\frac{1}{n}\Big]\,\text{Hypergeometric2F1}\Big[2,4,3+\frac{1}{n},\frac{(b c-a d)\,x^n}{c\,(a+b x^n)}\Big]\Big)\Big)+n^2\,x^n\,(c+d x^n)\,\Big(3\,a\,c^2\,(-b c+a d)\,(1+2n)\,(1+3n)\,(a+b x^n)\,\text{Gamma}\Big[2+\frac{1}{n}\Big]\,\text{Hypergeometric2F1}\Big[2,4,2+\frac{1}{n},\frac{(b c-a d)\,x^n}{c\,(a+b x^n)}\Big]-c\,d\,(1+3n)\,(a+b x^n)^2\,\Big(c\,(1+2n)\,(a+b x^n)\,\text{Gamma}\Big[2+\frac{1}{n}\Big]\,\text{Hypergeometric2F1}\Big[1,3,2+\frac{1}{n},\frac{(b c-a d)\,x^n}{c\,(a+b x^n)}\Big]+3\,(b c-a d)\,(1+n)\,x^n\,\text{Gamma}\Big[1+\frac{1}{n}\Big]\,\text{Hypergeometric2F1}\Big[2,4,3+\frac{1}{n},\frac{(b c-a d)\,x^n}{c\,(a+b x^n)}\Big]\Big)+3\,d\,(b c-a d)\,x^n\,\Big(b\,c\,(1+n)\,(1+3n)\,x^n\,(a+b x^n)\,\text{Gamma}\Big[1+\frac{1}{n}\Big]\,\text{Hypergeometric2F1}\Big[2,4,3+\frac{1}{n},\frac{(b c-a d)\,x^n}{c\,(a+b x^n)}\Big]-c\,(1+n)\,(1+3n)\,(a+b x^n)^2\,\text{Gamma}\Big[1+\frac{1}{n}\Big]\,\text{Hypergeometric2F1}\Big[2,4,3+\frac{1}{n},\frac{(b c-a d)\,x^n}{c\,(a+b x^n)}\Big]-a\,c\,n\,(1+3n)\,(a+b x^n)\,\text{Gamma}\Big[2+\frac{1}{n}\Big]\,\text{Hypergeometric2F1}\Big[2,4,3+\frac{1}{n},\frac{(b c-a d)\,x^n}{c\,(a+b x^n)}\Big]+8\,a\,(-b c+a d)\,n\,(1+n)\,x^n\,\text{Gamma}\Big[1+\frac{1}{n}\Big]\,\text{Hypergeometric2F1}\Big[3,5,4+\frac{1}{n},\frac{(b c-a d)\,x^n}{c\,(a+b x^n)}\Big]\Big)\Big)\Big)$$

Problem 237: Attempted integration timed out after 120 seconds.

$$\int \frac{(c+d x^n)^{2-\frac{1}{n}}}{(a+b x^n)^4}\,dx$$

Optimal (type 5, 133 leaves, 2 steps):

$$\frac{b\,x\,(c+d x^n)^{3-\frac{1}{n}}}{3\,a\,(b c-a d)\,n\,(a+b x^n)^3}-\frac{1}{3\,a^4\,(b c-a d)\,n}c^2\,(b c\,(1-3n)+3\,a\,d\,n)\,x\,(c+d x^n)^{-1/n}\,\text{Hypergeometric2F1}\Big[3,\frac{1}{n},1+\frac{1}{n},-\frac{(b c-a d)\,x^n}{a\,(c+d x^n)}\Big]$$

Result (type 1, 1 leaves):

???

Problem 242: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{-c+d x}\ \sqrt{c+d x}\ \left(a+b x^2\right)}{x^3}\, dx$$

Optimal (type 3, 96 leaves, 5 steps):

$$b \sqrt{-c+d x}\ \sqrt{c+d x}-\frac{a \sqrt{-c+d x}\ \sqrt{c+d x}}{2 x^2}-\frac{\left(2 b c^2-a d^2\right) \text{ArcTan}\left[\frac{\sqrt{-c+d x}\ \sqrt{c+d x}}{c}\right]}{2 c}$$

Result (type 3, 105 leaves):

$$\frac{1}{2} \left(\frac{\sqrt{-c+d x}\ \sqrt{c+d x}\ \left(-a+2 b x^2\right)}{x^2}+\left(2 i b c-\frac{i a d^2}{c}\right) \text{Log}\left[\frac{4 i c-4 \sqrt{-c+d x}\ \sqrt{c+d x}}{2 b c^2 x-a d^2 x}\right]\right)$$

Problem 243: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{-c+d x}\ \sqrt{c+d x}\ \left(a+b x^2\right)}{x^5}\, dx$$

Optimal (type 3, 121 leaves, 5 steps):

$$-\frac{\left(4 b c^2+a d^2\right) \sqrt{-c+d x}\ \sqrt{c+d x}}{8 c^2 x^2}+\frac{a (-c+d x)^{3/2} (c+d x)^{3/2}}{4 c^2 x^4}+\frac{d^2 \left(4 b c^2+a d^2\right) \text{ArcTan}\left[\frac{\sqrt{-c+d x}\ \sqrt{c+d x}}{c}\right]}{8 c^3}$$

Result (type 3, 132 leaves):

$$\frac{1}{8 c^3 x^4}\left(c \sqrt{-c+d x}\ \sqrt{c+d x}\ \left(-2 a c^2-4 b c^2 x^2+a d^2 x^2\right)-i d^2 \left(4 b c^2+a d^2\right) x^4 \text{Log}\left[\frac{16 c^2 \left(-i c+\sqrt{-c+d x}\ \sqrt{c+d x}\right)}{d^2 \left(4 b c^2+a d^2\right) x}\right]\right)$$

Problem 266: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b x^2}{x^3 \sqrt{-c+d x}\ \sqrt{c+d x}}\, dx$$

Optimal (type 3, 76 leaves, 3 steps):

$$\frac{a \sqrt{-c+d x}\ \sqrt{c+d x}}{2 c^2 x^2}+\frac{\left(2 b c^2+a d^2\right) \text{ArcTan}\left[\frac{\sqrt{-c+d x}\ \sqrt{c+d x}}{c}\right]}{2 c^3}$$

Result (type 3, 103 leaves):

$$\frac{a\,c\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}-i\,\left(2\,b\,c^2+a\,d^2\right)\,x^2\,\text{Log}\left[\frac{4\,c^2\,\left(-i\,c+\sqrt{-c+d\,x}\,\sqrt{c+d\,x}\right)}{\left(2\,b\,c^2+a\,d^2\right)\,x}\right]}{2\,c^3\,x^2}$$

Problem 268: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,x^2}{x^5\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}\,dx$$

Optimal (type 3, 123 leaves, 5 steps):

$$\frac{a\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{4\,c^2\,x^4}+\frac{\left(4\,b\,c^2+3\,a\,d^2\right)\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{8\,c^4\,x^2}+\frac{d^2\,\left(4\,b\,c^2+3\,a\,d^2\right)\,\text{ArcTan}\left[\frac{\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{c}\right]}{8\,c^5}$$

Result (type 3, 135 leaves):

$$\frac{1}{8\,c^5\,x^4}\left(c\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}\,\left(2\,a\,c^2+4\,b\,c^2\,x^2+3\,a\,d^2\,x^2\right)-i\,d^2\,\left(4\,b\,c^2+3\,a\,d^2\right)\,x^4\,\text{Log}\left[\frac{16\,c^4\,\left(-i\,c+\sqrt{-c+d\,x}\,\sqrt{c+d\,x}\right)}{d^2\,\left(4\,b\,c^2+3\,a\,d^2\right)\,x}\right]\right)$$

Problem 276: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,x^2}{x^3\,\left(-c+d\,x\right)^{3/2}\,\left(c+d\,x\right)^{3/2}}\,dx$$

Optimal (type 3, 117 leaves, 5 steps):

$$-\frac{2\,b\,c^2+3\,a\,d^2}{2\,c^4\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}+\frac{a}{2\,c^2\,x^2\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}-\frac{\left(2\,b\,c^2+3\,a\,d^2\right)\,\text{ArcTan}\left[\frac{\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{c}\right]}{2\,c^5}$$

Result (type 3, 126 leaves):

$$\frac{\frac{-2\,b\,c^3\,x^2+a\,\left(c^3-3\,c\,d^2\,x^2\right)}{x^2\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}+i\,\left(2\,b\,c^2+3\,a\,d^2\right)\,\text{Log}\left[\frac{4\,i\,c^5-4\,c^4\,\sqrt{-c+d\,x}\,\sqrt{c+d\,x}}{2\,b\,c^2\,x+3\,a\,d^2\,x}\right]}{2\,c^5}$$

Problem 278: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,x^2}{x^5\,\left(-c+d\,x\right)^{3/2}\,\left(c+d\,x\right)^{3/2}}\,dx$$

Optimal (type 3, 166 leaves, 7 steps):

$$-\frac{3 d^2 \left(4 b c^2+5 a d^2\right)}{8 c^6 \sqrt{-c+d x} \sqrt{c+d x}}+\frac{a}{4 c^2 x^4 \sqrt{-c+d x} \sqrt{c+d x}}+$$
$$\frac{4 b c^2+5 a d^2}{8 c^4 x^2 \sqrt{-c+d x} \sqrt{c+d x}}-\frac{3 d^2 \left(4 b c^2+5 a d^2\right) \text{ArcTan}\left[\frac{\sqrt{-c+d x} \sqrt{c+d x}}{c}\right]}{8 c^7}$$

Result (type 3, 157 leaves):

$$\frac{1}{8 c^7}\left(\frac{4 b c^3 x^2 \left(c^2-3 d^2 x^2\right)+a \left(2 c^5+5 c^3 d^2 x^2-15 c d^4 x^4\right)}{x^4 \sqrt{-c+d x} \sqrt{c+d x}}+\right.$$
$$\left.3 i \left(4 b c^2 d^2+5 a d^4\right) \text{Log}\left[\frac{16 i c^7-16 c^6 \sqrt{-c+d x} \sqrt{c+d x}}{12 b c^2 d^2 x+15 a d^4 x}\right]\right)$$

Problem 280: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^{-\frac{2 b^2 c+a^2 d}{b^2 c+a^2 d}} \left(c+d x^2\right)}{\sqrt{-a+b x} \sqrt{a+b x}} \, dx$$

Optimal (type 3, 53 leaves, 1 step):

$$\left(\frac{c}{a^2}+\frac{d}{b^2}\right) x^{-\frac{b^2 c}{b^2 c+a^2 d}} \sqrt{-a+b x} \sqrt{a+b x}$$

Result (type 6, 1424 leaves):

$$-\frac{1}{b^4 \sqrt{-a+b x} \sqrt{a+b x} \sqrt{1-\frac{b^2 x^2}{a^2}}} d \left(b^2 c+a^2 d\right) x^{-\frac{b^2 c}{b^2 c+a^2 d}}$$
$$\left(-\frac{1}{c} (a-b x) (a+b x) \text{Hypergeometric2F1}\left[-\frac{1}{2},-\frac{b^2 c}{2 \left(b^2 c+a^2 d\right)},1-\frac{b^2 c}{2 \left(b^2 c+a^2 d\right)},\frac{b^2 x^2}{a^2}\right]+\right.$$
$$\left(a b^2 (a-b x)^2 \sqrt{1+\frac{b x}{a}} \text{AppellF1}\left[-\frac{b^2 c}{b^2 c+a^2 d},-\frac{1}{2},\frac{1}{2},\frac{a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]\right) \Big/$$
$$\left(\sqrt{1-\frac{b x}{a}} \left(2 a^3 d \text{AppellF1}\left[-\frac{b^2 c}{b^2 c+a^2 d},-\frac{1}{2},\frac{1}{2},\frac{a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]-\right.\right.$$
$$b \left(b^2 c+a^2 d\right) x \left(\text{AppellF1}\left[\frac{a^2 d}{b^2 c+a^2 d},-\frac{1}{2},\frac{3}{2},\frac{b^2 c+2 a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]+\right.$$
$$\left.\left.\left.\left.\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{a^2 d}{2 \left(b^2 c+a^2 d\right)}\right\},\left\{\frac{b^2 c}{b^2 c+a^2 d}+\frac{3 a^2 d}{2 \left(b^2 c+a^2 d\right)}\right\},\frac{b^2 x^2}{a^2}\right]\right)\right)\right)\right)+$$

$$\left(a^3 d\, (a-b x)^2 \sqrt{1+\frac{b x}{a}}\ \text{AppellF1}\left[-\frac{b^2 c}{b^2 c+a^2 d},-\frac{1}{2},\frac{1}{2},\frac{a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]\right)\Big/$$
$$\left(c \sqrt{1-\frac{b x}{a}} \left(2 a^3 d\, \text{AppellF1}\left[-\frac{b^2 c}{b^2 c+a^2 d},-\frac{1}{2},\frac{1}{2},\frac{a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]-\right.\right.$$
$$b \left(b^2 c+a^2 d\right) x \left(\text{AppellF1}\left[\frac{a^2 d}{b^2 c+a^2 d},-\frac{1}{2},\frac{3}{2},\frac{b^2 c+2 a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]+\right.$$
$$\left.\left.\left.\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{a^2 d}{2 \left(b^2 c+a^2 d\right)}\right\},\left\{\frac{b^2 c}{b^2 c+a^2 d}+\frac{3 a^2 d}{2 \left(b^2 c+a^2 d\right)}\right\},\frac{b^2 x^2}{a^2}\right]\right)\right)\right)+$$
$$\left(a b^2 (a+b x)^2 \sqrt{1-\frac{b x}{a}}\ \text{AppellF1}\left[-\frac{b^2 c}{b^2 c+a^2 d},\frac{1}{2},-\frac{1}{2},\frac{a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]\right)\Big/$$
$$\left(\sqrt{1+\frac{b x}{a}} \left(2 a^3 d\, \text{AppellF1}\left[-\frac{b^2 c}{b^2 c+a^2 d},\frac{1}{2},-\frac{1}{2},\frac{a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]+\right.\right.$$
$$b \left(b^2 c+a^2 d\right) x \left(\text{AppellF1}\left[\frac{a^2 d}{b^2 c+a^2 d},\frac{3}{2},-\frac{1}{2},\frac{b^2 c+2 a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]+\right.$$
$$\left.\left.\left.\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{a^2 d}{2 \left(b^2 c+a^2 d\right)}\right\},\left\{\frac{b^2 c}{b^2 c+a^2 d}+\frac{3 a^2 d}{2 \left(b^2 c+a^2 d\right)}\right\},\frac{b^2 x^2}{a^2}\right]\right)\right)\right)+$$
$$\left(a^3 d\, (a+b x)^2 \sqrt{1-\frac{b x}{a}}\ \text{AppellF1}\left[-\frac{b^2 c}{b^2 c+a^2 d},\frac{1}{2},-\frac{1}{2},\frac{a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]\right)\Big/$$
$$\left(c \sqrt{1+\frac{b x}{a}} \left(2 a^3 d\, \text{AppellF1}\left[-\frac{b^2 c}{b^2 c+a^2 d},\frac{1}{2},-\frac{1}{2},\frac{a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]+\right.\right.$$
$$b \left(b^2 c+a^2 d\right) x \left(\text{AppellF1}\left[\frac{a^2 d}{b^2 c+a^2 d},\frac{3}{2},-\frac{1}{2},\frac{b^2 c+2 a^2 d}{b^2 c+a^2 d},\frac{b x}{a},-\frac{b x}{a}\right]+\right.$$
$$\left.\left.\left.\left.\text{HypergeometricPFQ}\left[\left\{\frac{1}{2},\frac{a^2 d}{2 \left(b^2 c+a^2 d\right)}\right\},\left\{\frac{b^2 c}{b^2 c+a^2 d}+\frac{3 a^2 d}{2 \left(b^2 c+a^2 d\right)}\right\},\frac{b^2 x^2}{a^2}\right]\right)\right)\right)\right)$$

Problem 281: Unable to integrate problem.

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\ \sqrt{-1+\sqrt{x}}\ \sqrt{1+x}}\,dx$$

Optimal (type 3, 36 leaves, 3 steps):

$$\frac{\sqrt{1-x}\ \text{ArcSin}[x]}{\sqrt{-1-\sqrt{x}}\ \sqrt{-1+\sqrt{x}}}$$

Result (type 8, 34 leaves):

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\ \sqrt{-1+\sqrt{x}}\ \sqrt{1+x}}\,dx$$

Problem 282: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\ \sqrt{a+b\sqrt{x}}\ \sqrt{a^2+b^2 x}}\,dx$$

Optimal (type 3, 75 leaves, 4 steps):

$$-\frac{2\sqrt{a^2-b^2 x}\ \text{ArcTan}\left[\frac{\sqrt{a^2-b^2 x}}{\sqrt{a^2+b^2 x}}\right]}{b^2\sqrt{a-b\sqrt{x}}\ \sqrt{a+b\sqrt{x}}}$$

Result (type 8, 43 leaves):

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\ \sqrt{a+b\sqrt{x}}\ \sqrt{a^2+b^2 x}}\,dx$$

Problem 283: Unable to integrate problem.

$$\int \left(a-b x^n\right)^p \left(a+b x^n\right)^p \left(c+d x^{2n}\right)^q\,dx$$

Optimal (type 6, 113 leaves, 4 steps):

$$x\left(a-b x^n\right)^p \left(a+b x^n\right)^p \left(1-\frac{b^2 x^{2n}}{a^2}\right)^{-p} \left(c+d x^{2n}\right)^q \left(1+\frac{d x^{2n}}{c}\right)^{-q} \text{AppellF1}\left[\frac{1}{2n},\ -p,\ -q,\ \frac{1}{2}\left(2+\frac{1}{n}\right),\ \frac{b^2 x^{2n}}{a^2},\ -\frac{d x^{2n}}{c}\right]$$

Result (type 8, 33 leaves):

$$\int \left(a-b x^n\right)^p \left(a+b x^n\right)^p \left(c+d x^{2n}\right)^q\,dx$$

Problem 284: Unable to integrate problem.

$$\int \left(a-b x^n\right)^p \left(a+b x^n\right)^p \left(a^2+b^2 x^{2n}\right)^p\,dx$$

Optimal (type 5, 87 leaves, 4 steps):

$$x\left(a-b x^n\right)^p\left(a+b x^n\right)^p\left(a^2+b^2 x^{2n}\right)^p\left(1-\frac{b^4 x^{4n}}{a^4}\right)^{-p}\text{Hypergeometric2F1}\left[\frac{1}{4n},-p,\frac{1}{4}\left(4+\frac{1}{n}\right),\frac{b^4 x^{4n}}{a^4}\right]$$

Result (type 8, 37 leaves):

$$\int\left(a-b x^n\right)^p\left(a+b x^n\right)^p\left(a^2+b^2 x^{2n}\right)^p\,dx$$

Problem 285: Result more than twice size of optimal antiderivative.

$$\int\frac{\left(c+d x^{2n}\right)^p}{\left(a-b x^n\right)\left(a+b x^n\right)}\,dx$$

Optimal (type 6, 76 leaves, 3 steps):

$$\frac{1}{a^2}x\left(c+d x^{2n}\right)^p\left(1+\frac{d x^{2n}}{c}\right)^{-p}\text{AppellF1}\left[\frac{1}{2n},1,-p,\frac{1}{2}\left(2+\frac{1}{n}\right),\frac{b^2 x^{2n}}{a^2},-\frac{d x^{2n}}{c}\right]$$

Result (type 6, 258 leaves):

$$\left(a^2 c\left(1+2n\right)x\left(c+d x^{2n}\right)^p\text{AppellF1}\left[\frac{1}{2n},-p,1,1+\frac{1}{2n},-\frac{d x^{2n}}{c},\frac{b^2 x^{2n}}{a^2}\right]\right)\Big/$$
$$\left(\left(a^2-b^2 x^{2n}\right)\left(2a^2 d\,n\,p\,x^{2n}\text{AppellF1}\left[1+\frac{1}{2n},1-p,1,2+\frac{1}{2n},-\frac{d x^{2n}}{c},\frac{b^2 x^{2n}}{a^2}\right]+\right.\right.$$
$$2b^2 c\,n\,x^{2n}\text{AppellF1}\left[1+\frac{1}{2n},-p,2,2+\frac{1}{2n},-\frac{d x^{2n}}{c},\frac{b^2 x^{2n}}{a^2}\right]+$$
$$\left.\left.a^2 c\left(1+2n\right)\text{AppellF1}\left[\frac{1}{2n},-p,1,1+\frac{1}{2n},-\frac{d x^{2n}}{c},\frac{b^2 x^{2n}}{a^2}\right]\right)\right)$$

Problem 286: Unable to integrate problem.

$$\int\left(a-b x^{n/2}\right)^p\left(a+b x^{n/2}\right)^p\left(\frac{a^2 d\left(1+p\right)}{b^2\left(1+\frac{-1-2n-n p}{n}\right)}+d x^n\right)^{\frac{-1-2n-n p}{n}}dx$$

Optimal (type 3, 96 leaves, 2 steps):

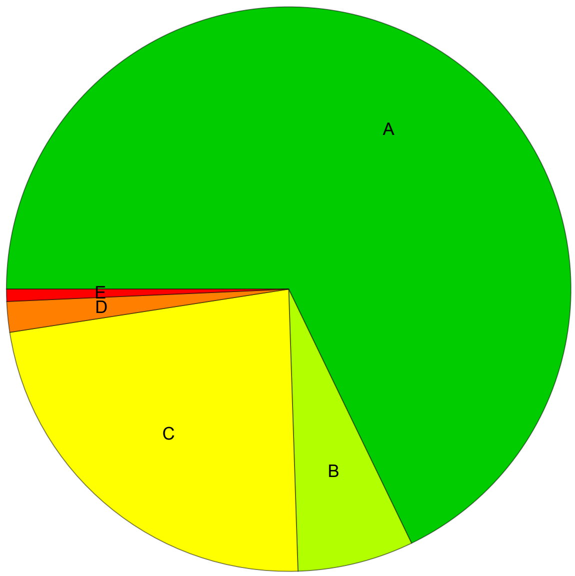
$$-\frac{b^2\left(1+n+n p\right)x\left(a-b x^{n/2}\right)^{1+p}\left(a+b x^{n/2}\right)^{1+p}\left(-\frac{a^2 d n\left(1+p\right)}{b^2\left(1+n+n p\right)}+d x^n\right)^{-\frac{1+n+n p}{n}}}{a^4 d n\left(1+p\right)}$$

Result (type 8, 78 leaves):

$$\int\left(a-b x^{n/2}\right)^p\left(a+b x^{n/2}\right)^p\left(\frac{a^2 d\left(1+p\right)}{b^2\left(1+\frac{-1-2n-n p}{n}\right)}+d x^n\right)^{\frac{-1-2n-n p}{n}}dx$$

Summary of Integration Test Results

286 integration problems



A - 194 optimal antiderivatives

B - 19 more than twice size of optimal antiderivatives

C - 66 unnecessarily complex antiderivatives

D - 5 unable to integrate problems

E - 2 integration timeouts